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AN EXPERIMENTAL RESEARCH ON THE LONGITUDINAL AND TORSIONAL ELASTIC FATIGUE.

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THE fact that, when an elastic body is subjected to a strain, there is a deformation of the body, which does not entirely disappear for some time after the strain is removed, even when the limit of elasticity is not passed, was first noticed by W. Weber,[1] who named the phenomenon the *Elastische Nachwirkung* (elastic fatigue). Weber's work was done on silk fibers. Later, Kohlrausch[2] investigated glass, rubber, and wires of different material, and proved that the elastic fatigue follows the same laws in all these substances at least for torsional strain, and in the case of rubber also for stretching and bending. I give below the principal results of his investigation.

I. If a body is subjected to a strain, the resulting fatigue is nearly proportional to that strain.

II. If the strains continue for different lengths of time, the fatigue is proportional to a power of the time.

III. The fatigue increases uniformly with the temperature.

IV. The curves, which represent the return of a body to its original shape after being subjected to different strains for equal lengths of time, are similar.

[1] W. Weber: Pogg. Ann., XXXIV., p. 247; LIV., p. 1.

[2] F. Kohlrausch: Pogg. Ann., CXIX., p. 337; CXXVIII., pp. 7–207–399; CLVIII., p. 337; CLX., p. 225.

V. In all these cases the fatigue may be represented by the following formulæ: —

1. $$-\frac{dx}{dt}=\alpha\frac{x}{t^n}, \qquad x=Ce^{-\alpha t^m},$$

where $\alpha=am$, and $n=1-m$.

When the time of strain did not exceed three minutes, the more simple form ($n=1$) could be used: —

2. $$-\frac{dx}{dt}=\alpha\frac{x}{t}, \qquad x=\frac{c}{t^\alpha}.$$

Here t is the time which has passed since the release from strain. The distance from the final position of equilibrium is represented by x. n, c, C, and α are constants.

A theoretical formula for the fatigue was derived by Boltzmann, who assumed that a strain, δ, which had continued during a time, $d\theta$, left behind it a fatigue that was proportional to $\delta d\theta$, and a function of the time, θ, which had passed since the release from strain.

One form of the hypothesis developed by Boltzmann[1] supposes that the fatigue is proportional to $\delta\frac{d\theta}{\theta}$. That is, if a body has undergone a strain during a time t, after a time t, counting from the moment of release from strain, there still exists a deformation, x, which, if p is a constant of the material, equals

$$x=p\delta\int_t^{t+T}\frac{d\theta}{\theta}=p\delta\log\frac{t+T}{t}.$$

Observations on the longitudinal fatigue of metals have been made by Miller,[2] but not in such a manner as to allow the study of the whole course of the phenomenon. I have therefore thought it of interest to carry out a series of observations on this form of fatigue on different kinds of wires, and also to investigate their torsional fatigue.

[1] Boltzmann: Pogg. Ann. Erg., Bd. VII., p. 624.

[2] Miller: Sitz. Berichte des K. bayr. Akademie, 1878.

The chief questions are as follows: —

I. Does the longitudinal fatigue follow the same laws as the torsional?

II. How does the velocity of return to the position of equilibrium depend on the duration of the original strain?

III. Are the curves, which represent the course of the fatigue during strain and after release from strain, congruent?

IV. What relation exists between the longitudinal and torsional fatigue?

Apparatus.

The portion of the investigation on the longitudinal fatigue was carried out in the tower of the Physical Laboratory in Strasburg, which is high enough to allow wires 23 m. long to be used, and as it lies on the north side of the building, and is built with very heavy stone walls, the temperature retains a very admirable degree of constancy. The tower contains four piers, entirely free from the floors, and extending nearly to its top. The wires used were fastened by iron clamps to a beam 25 cm. square, which was laid across two of these, so as to form a perfectly solid support. To be sure that the wires were held fast, a microscope was placed on a second beam alongside of the first, through which a mark on the wire was frequently observed.

To eliminate the effects of changes of temperature two wires of the same material were used. One of these bore a constant weight and also the mark, which was taken as the zero in the observations. Close to this standard wire, and running through the hollow weight, which was suspended from it, was hung the wire on which the fatigue was observed. On the bottom of this wire was fastened the pan for the weights, and below this was an ordinary large water damper. To prevent any tendency to revolve, each wire was provided with a light arm, which played with great freedom between two glass rods fastened to the pier. The observations were taken by means of a microscope provided with a micrometer scale in the eye-piece, which permitted readings to be taken to 0.001 mm. The edges of glass fibers fastened to the wires with sealing wax were used as marks. To prevent lateral motion,

a little below the marks, each wire was allowed to rest without observable friction in the angle formed by two stiff glass fibers, which were fastened into a frame attached to the pier. The temperature of the tower was determined by observing the expansion and contraction of the standard wire through a second microscope.

The investigation on the torsional fatigue was carried out in another room of the laboratory. The wire hung in a case provided with a graduated torsion head, and bore at its lower end a mirror and a small stretching weight, to which was attached a water damper. An arm extended out from the weight between two vertical wires, thus preventing too great amplitude of swinging, and holding the lower end of the wire in place, when the torsion was applied. The observations were carried out in the ordinary way with telescope and scale.

General Method of the Research.

The Longitudinal Fatigue. — After each wire was hung in its place, it was stretched for twenty-four hours by a weight twice as great as any used in the experiments, and was then left for from three days to a week, after which the observations were begun. The first observation of each series was taken one-half minute after the removal of the weight; but as the wire had often not become perfectly steady at that time, too great weight is not to be attached to these values, and on this account they were not used in the calculation of the constants.

The Torsional Fatigue. — The wires for the torsional observations were hung and subjected to large torsions, with periods of rest between, for several days, and in some cases weeks, before they were finally used in the investigation. The one-half minute observations were here also not satisfactory, as the mirror was always in motion at that time. After each series of observations, no new ones were begun until the old fatigue had entirely disappeared, or until there had been no apparent movement of the wire for a time equal to that required for the coming series.

The meaning of the symbols used in this paper are as follows:—

τ denotes the temperature.
W " the stretching weight.
L " the resulting elongation.
ϕ " the angle of torsion.
T " the time of strain.
t " the time, counting from the release from strain.
x " the fatigue.
x_1 " the fatigue for $(t = 1)$.[1]
Δ " x observed $- x$ calculated.

λ equals $\frac{x_1}{L}$ for the elongation.

λ' equals $\frac{x_1}{\phi}$ for the torsion.[2]

Brass Wire.

Longitudinal Fatigue.

Length	23 m.
Diameter	0.30 mm.
Breaking weight	4.7 kg.
Constant stretching weight . . .	0.65 kg.
Elongation for 1 kg.	36 mm.

Table I. represents a number of series of observations, showing the fatigue resulting from different degrees of tension for equal lengths of time, and also that of equal tensions for different times. The unit chosen in the case of the longitudinal fatigue is 0.01 mm. The first three series show the fatigue due to a load of 0.5 kg., 0.7 kg., and 1 kg. applied for three minutes.

[1] The calculated value of x_1 equals the constant c of the formula.
[2] x_1 and L, x_1 and ϕ are of course expressed respectively in the same units.

TABLE I.

$T = 3$ min.

	$W = 0.5$ kg. $L = 1800$ $\tau = 20°.7$			$W = 0.7$ kg. $L = 2520$ $\tau = 19°.3$			$W = 1$ kg. $L = 3600$ $\tau = 19°.7$		
t	x		Δ	x		Δ	x		Δ
Min.	Obs.	Cal.		Obs.	Cal.		Obs.	Cal.	
0.5	2.0	2.16	−0.16	3.2	3.43	−0.23	4.6	4.85	−0.25
1.0	1.6	1.55	+0.05	2.4	2.46	−0.06	3.2	3.48	−0.28
2.0	1.2	1.11	+0.09	1.8	1.76	+0.04	2.6	2.50	+0.10
3.0	1.0	1.91	+0.09	1.5	1.45	+0.05	2.2	2.05	+0.15
5.0	0.7	0.72	−0.02	1.2	1.14	+0.06	1.8	1.61	+0.19
7.0	0.5	0.61	−0.11	1.0	0.97	+0.03	1.4	1.37	+0.03
10.0	0.3	0.51	−0.21	0.8	0.81	−0.01	1.2	1.15	+0.05
15.0				0.6	0.67	−0.07	0.9	0.95	−0.05
20.0				0.5	0.58	−0.08	0.8	0.83	−0.03
	$c = 1.55$			$c = 2.46$			$c = 3.48$		
	$a = 0.48$					$x = \frac{c}{t^a}$			

$W = 1$ kg. $L = 3600$

	$T = 27$ min. $\tau = 20°$			$T = 81$ min. $\tau = 19°.7$		
t	x		Δ	x		Δ
Min.	Obs.	Cal.		Obs.	Cal.	
1	10.2	10.13	+0.07	16.0	15.89	+0.11
2	8.8	8.78	+0.02	13.8	14.06	−0.26
3	7.8	7.96	−0.16	12.8	12.92	−0.12
5	6.8	6.91	−0.11	11.4	11.44	−0.04
7	6.1	6.21	−0.11	10.6	10.44	+0.16
10	5.4	5.48	−0.08	9.2	9.38	−0.18
15	4.6	4.67	−0.07	8.2	8.18	+0.02
20	4.0	4.12	−0.12	7.4	7.34	+0.06
30	3.4	3.39	+0.01	6.4	6.21	+0.19
40	3.2	2.90	+0.30			
50	2.8	2.55	+0.25	5.0	4.87	+0.13
70	2.0	2.06	−0.06			
80				3.8	3.75	+0.05
110				3.2	3.08	+0.12
	$C = 18.80$ $a = 0.618$			$C = 27$ $a = 0.530$		
	$m = 0.3$			$x = Ce^{-at^m}$		

The calculated[1] values were derived according to Kohlrausch's formula 2 (p. 1), which he found to apply in the case of torsion, when the strain had not lasted more than three minutes. The calculated curves are exactly similar, being calculated with the same value of α in all three series. The similarity of the observed curves is in general good, though from the values of Δ it is seen that for a greater load the curve descends a little more slowly than for a smaller one. For greater lengths of time of strain, formula 2 no longer applies, and the general formula must be used. In all these points, the longitudinal fatigue agrees with the torsional, as studied by Kohlrausch.

Kohlrausch found further that for a given temperature the value of x_1 is nearly proportional to the deforming force, and also to a power of the time T; or,

$$x_1 = KWT^{\beta},$$

when W is the deforming force, and K and β are constants. To prove this formula for the longitudinal fatigue, I have taken the observations from Table I., and also some others which are not elsewhere given, and have collected them in Table II. All the observations are reduced to a temperature of 20° (see p. 8).

If we let

$$K = 2.13,\ \beta = 0.46,$$

the formula becomes

$$x_1 = 2.13 \cdot W \cdot T^{0.46}.$$

TABLE II.

W	T	x_1		Δ
		Obs.	Cal.	
1 kg.	1 min.	2.14	2.13	+0.01
1 "	3 "	3.23	3.53	−0.30
1 "	9 "	5.80	5.85	−0.05
1 "	27 "	10.20	9.70	+0.50
1 "	81 "	16.12	16.08	+0.04
0.7	3	2.44	2.47	−0.03
0.5	3	1.58	1.76	−0.18

[1] The constants were derived by the method of least squares

The elongation of the wire for a weight of 1 kg. is 36 mm.

If we replace the weight W by the elongation L, and remember that x_1 is expressed in 0.01 mm. for a temperature of 20°, and $T=1$ min.,

$$\frac{x_1}{L} = 0.00059 = \lambda.$$

Torsional Fatigue.

I give only a few observations of the torsional fatigue, as the subject is already well understood. Table III. shows the results of three series of observations on the course of the fatigue for different degrees of torsion. ϕ denotes the angle of torsion, otherwise the symbols are the same as in Table I.

TABLE III.

BRASS TORSION.

	$T=2$ min.								
	$\phi=30°$ $\tau=23°.3$			$\phi=60°$ $\tau=23°2$			$\phi=90°$ $\tau=23°6$		
t	x		Δ	x		Δ	x		Δ
	Obs.	Cal.		Obs.	Cal.		Obs.	Cal.	
0.5	4.0	4.50	−0.50	8.0	8.56	−0.56	12.8	13.23	−0.43
1.0	2.9	2.99	−0.09	5.5	5.69	−0.19	8.5	8.79	−0.29
2.0	2.1	1.99	+0.11	4.0	3.78	+0.22	6.0	5.84	+0.16
3.0	1.8	1.56	+0.24	3.1	2.98	+0.12	4.8	4.60	+0.20
5.0	1.0	1.16	−0.16	2.4	2.20	+0.20	3.6	3.40	+0.20
7.0	0.9	0.95	−0.05	2.0	1.81	+0.19	2.9	2.79	+0.11
10.0	0.8	0.77	+0.03	1.3	1.46	−0.16	2.1	2.26	−0.16
15.0				1.1	1.15	−0.05	1.8	1.78	+0.02
20.0				0.9	0.97	−0.07	1.5	1.50	±0.00
	$c=2.99$			$c=5.69$			$c=8.79$		
	$\alpha=0.59$			$x=\frac{c}{t^{\alpha}}$					

The similarity of the curves is here evident, and the agreement between the observed and calculated values is, with the exception of those for $t=\frac{1}{2}$ min., fairly good. x is here given in scale divisions which are equal to 0.018° in circular measure.

The value of x_1 for $T=1$ min. and $\tau=20°$ expressed in terms of the angle of torsion,

$$0.018\frac{x_1}{\phi}=0.0011=\lambda'.$$

Comparing this with the corresponding result for the longitudinal fatigue, we find

$$\lambda'=1.9\lambda.$$

The Influence of the Temperature.

The rôle played by the temperature is an important one, and it is to be regretted that it was found impossible to make a complete series of observations of its effects in the case of the longitudinal fatigue, but as there is no way of warming the tower, and the change from summer to winter temperature came while I was away from Strasburg, I was only able to make observations at these two temperatures. However, these two indicate that the temperature effect is nearly the same as in the case of the torsion.

Table IV. gives the values of x_1 for $\phi=120°$ and $T=1$ min. at four, and also for $W=1$ kg. and $T=3$ min. at two different temperatures.

TABLE IV.

Torsion		Tension	
τ	x_1	τ	x_1
23°	7.8	20°	3.3
19°	7.1		
16°	6.7		
11°	5.8	10°	2.4

The observations show the value of the torsion temperature coefficient, or increase of fatigue for 1° of temperature, in the neighborhood of 15°, to be

$$x=0.029.$$

The value for the longitudinal coefficient from these observations is 0.037. The observations in the last case are perhaps not exact enough to prove that the difference really exists.

The influence of change of temperature on the constant α, if it exists at all, is extremely small, but some of my observations would indicate that α decreases slightly with increasing temperature.

Fatigue of Long-continued Longitudinal Strain.

The accurate observation of the fatigue of this sort is extremely difficult, since, contrary to the usual views on elasticity, even a small weight, long enough applied, seems sufficient to produce a permanent set.

From this cause any positive conclusions in regard to the simi-

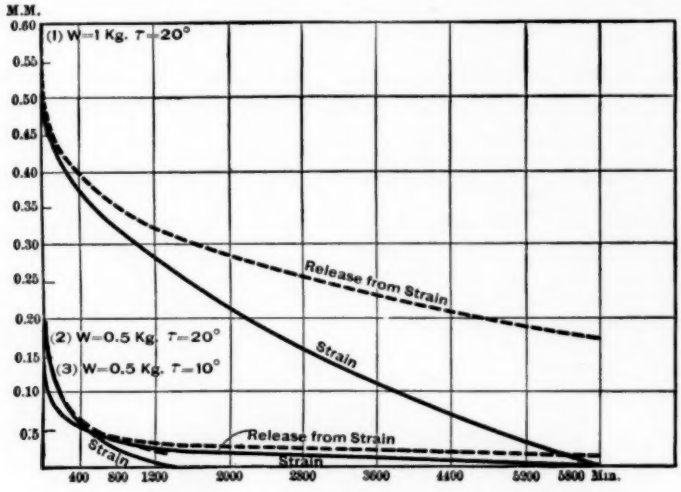


Fig. I.

larity of the curves for different degrees of strain, or during strain and after release from strain, is impossible.

Kohlrausch[1] found during his observations on rubber, that in this last case the curves became more and more nearly congruent, as the stretching weights were made smaller, so that it seemed probable that for infinitely small strains they would really be of the same form.

In Fig. I. are given three pairs of curves, the solid line in each case representing the course of the fatigue during strain, and the dotted line after the release from strain.

[1] F. Kohlrausch, Pogg. Ann., Bd. CLVIII., p. 337.

The curves (1) and (2) were taken at a temperature of 20°, while (3) was at 10°. The differences in the two parts of (1) can undoubtedly be in part explained by the permanent set of the wire.

The similarity in the parts of (2) and (3) is much greater, and it seems quite probable that, if there were no permanent set, the two parts would coincide in both cases.

Copper Wire.

Longitudinal Fatigue.

Length	23 m.
Diameter	0.29 mm.
Breaking weight	2.3 kg.
Constant stretching weight	0.44 kg.
Elongation for 1 kg.	28 mm.

Though copper gives a greater amount of fatigue for a given strain than the other metals investigated, its low limit of elasticity made it necessary to use smaller weights, and therefore the actual results are in general less.

Table V.

$T = 0.5$ min.

t	$W = 0.25$ kg. $L = 700$ $\tau = 18°$			$W = 0.5$ kg. $L = 1400$ $\tau = 20°.3$			$W = 0.6$ kg. $L = 1680$ $\tau = 18°.6$		
	x		Δ	x		Δ	x		Δ
	Obs.	Cal.		Obs.	Cal.		Obs.	Cal.	
0.5	1.2	0.90	+0.30	2.8	2.53	+0.27	4.0	3.47	+0.53
1.0	0.8	0.71	+0.09	2.0	2.00	±0.00	2.6	2.74	−0.14
2.0	0.6	0.56	+0.04	1.6	1.58	+0.02	2.2	2.16	+0.04
3.0	0.5	0.49	+0.01	1.4	1.38	+0.02	2.0	1.88	+0.12
5.0	0.3	0.41	−0.11	1.2	1.16	+0.04	1.6	1.58	+0.02
7.0				1.0	1.03	−0.03	1.4	1.41	−0.01
10.0	0.2	0.33	−0.13	0.9	0.92	−0.02	1.3	1.25	+0.05
15.0				0.8	0.80	±0.00	1.1	1.09	+0.01
20.0				0.7	0.74	−0.04	1.0	1.01	−0.01
	$\epsilon = 0.713$			$\epsilon = 2.002$			$\epsilon = 2.737$		

$\alpha = 0.34$

$x = \frac{\epsilon}{f\alpha}$

TABLE V. (*continued.*)

	$W = 0.5$ kg. $L = 1400$					
	$T = 3$ min. $\tau = 10°$			$T = 20$ min. $\tau = 10°.1$		
t	x		Δ	x		Δ
	Obs.	Cal.		Obs.	Cal.	
1	3.6	3.67	−0.07	7.7	7.67	+0.03
2	3.2	3.10	+0.10	6.7	6.61	+0.09
3	2.8	2.76	+0.04	6.0	5.99	+0.01
5	2.3	2.33	−0.03	5.3	5.21	+0.09
7	2.0	2.05	−0.05	4.6	4.69	−0.09
10	1.8	1.75	+0.05	4.0	4.15	−0.15
15	1.4	1.42	−0.02	3.5	3.53	−0.03
20	1.1	1.18	−0.08	3.0	3.09	−0.09
30	0.9	0.84	+0.06	2.5	2.47	+0.03
50				1.7	1.69	+0.01
70				1.3	1.17	+0.13
	$c = 3.67$			$c = 7.67$		
	$p = 0.001370$			$p = 0.002515$		
	$x = c - p\,L \log t$					

Table V. shows the fatigue resulting for weights of 0.25 kg., 0.5 kg., and 0.6 kg., applied for 0.5 min., and also for 0.5 kg. for 3 min. and 20 min. The first three series are calculated from Kohlrausch's formula 2, and the agreement between the observed and calculated values leaves nothing to be desired. a is here somewhat smaller than in brass.

The fatigue in copper increases more rapidly in proportion than the weight W; this relation may be expressed for $T = 0.5$ min., and $\tau = 20°$ by the equation :

$$x_1 = 2.62\ W + 2.97\ W^2$$
$$= 0.00094\ L + 0.00000038\ L^2.$$

From this we get

TABLE VI. *a*.

W	x_1		Δ
	Obs.	Cal.	
0.25	0.84	0.84	±0.00
0.50	1.99	2.05	−0.06
0.60	2.68	2.64	+0.04

The variations of x_1 for $W=0.5$ kg. and $\tau=20°$, due to different durations of strain, are given in the following table:—

TABLE VI. *b*.

T	x_1		Δ
	Obs.	Cal.	
0.5	1.99	2.00	−0.01
1.0	2.70	2.71	−0.01
3.0	4.68	4.39	+0.29
20.0	10.01	10.11	−0.10

Here $x_1 = 2.71\, T^{0.44}$

Connecting the results of Tables VI. *a* and VI. *b* as a general expression, we may write

$$x_1 = (0.00127\, L + 0.0000005 L^2)\, T^{0.44}.$$

The last two series of Table V. I have attempted to calculate according to Boltzmann's formula:

$$x = pL \log \frac{t+T}{t}.$$

This, however, did not give satisfactory results. I then tried the more simple formula,

$$x = c - pL \log t.$$

With this the agreement between the observed and calculated values proved to be good.

Torsional Fatigue.

TABLE VII.

COPPER TORSION.

$T = 1$ min.

	$\phi = 20°$ $\tau = 20°.2$			$\phi = 30°$ $\tau = 19°.9$			$\phi = 40$ $\tau = 18°6$		
t	x		Δ	x		Δ	x		Δ
	Obs.	Cal.		Obs.	Cal.		Obs.	Cal.	
0.5	11.5	11.19	+0.31	18.0	17.57	+0.43	25.5	24.79	+0.71
1.0	7.9	7.86	+0.04	12.5	12.34	+0.16	17.6	17.41	+0.19
2.0	5.4	5.52	−0.12	8.8	8.66	+0.14	12.5	12.23	+0.27
3.0	4.4	4.49	−0.09	6.9	7.04	−0.14	9.8	9.94	−0.14
5.0	3.4	3.46	−0.06	5.1	5.43	−0.33	7.4	7.66	−0.26
7.0	2.9	2.91	−0.01	4.4	4.57	−0.17	6.2	6.45	−0.25
10.0	2.5	2.43	+0.07	3.8	3.81	−0.01	5.2	5.38	−0.18
15.0	2.1	1.98	+0.12	3.2	3.10	+0.10	4.3	4.37	−0.07
20.0	1.9	1.71	+0.19	2.8	2.68	+0.12	3.5	3.78	−0.28
		$c = 7.86$			$c = 12.34$			$c = 17.41$	

$\alpha = 0.51$

$x = \frac{c}{t^{\alpha}}$

Table VII., while showing that the curves of the fatigue are nearly similar, indicates that for larger torsions the return to equilibrium is slightly more rapid than for smaller.

The value for α is larger than in the longitudinal fatigue, but not as large as the corresponding α of brass. x_1 increases more rapidly in proportion than the angle of torsion, and in a quadratic expression its value is

$$x_1 = 0.324\,\phi + 0.0033\,\phi^2;$$

or, expressed in degrees, each scale division being equal to 0.014°,

$$0.014\,x_1 = 0.00454\,\phi + 0.000046\,\phi^2.$$

For small torsions applied for 1 min., at a temperature of 20°,

$$\lambda' = 0.014\frac{x_1}{\phi} = 0.0045.$$

For the longitudinal fatigue the corresponding value of λ was 0.00127; then

$$\lambda' = 3.6\,\lambda.$$

The fact that the fatigue does not increase at the same rate as the strain which produces it in copper, makes this last comparison unsatisfactory.

The Influence of the Temperature.

The values of x_1 for $\phi = 20°$ and $T = 1$ min., and also for $W = 0.5$ kg. and $T = 0.5$ min., are given in Table VIII.

TABLE VIII.

τ	x_1	
	Torsion.	Tension.
20°	7.8	2.0
15°	6.8	—
12°	6.2	—
10°	—	1.6
	$\gamma = 0.034$	$\gamma = 0.025$

Fatigue of Long-continued Longitudinal Strain.

Copper, from its extreme liability to take a permanent set during long-continued tension, was even more difficult to investigate than brass; but the conclusions which I am able to draw from the observations are somewhat more satisfactory.

Fig. II. shows two pairs of curves, the solid line, as before, representing the course of the fatigue during strain, and the dotted line after release from strain.

No. 1 ($W = 0.3$ kg.; $T = 5500$ min.; average temperature, $\tau = 10°.2$) shows a permanent set equal to nearly a quarter of the height of the whole curve, and it is manifestly impossible to make any comparison between the two parts.

No. 2 ($W = 0.15$ kg.; $T = 2800$ min.; average temperature, $\tau = 10°.5$) has a permanent set of about one-sixteenth of its total

height. That this is really a set, and not due to a slow rate of return to a position of equilibrium, I am sure, as I observed the wire for more than a week after the last-recorded observation was taken; and the mark never varied by as much as 0.001 mm. This is also true for curve No. 1, though in this case the observations

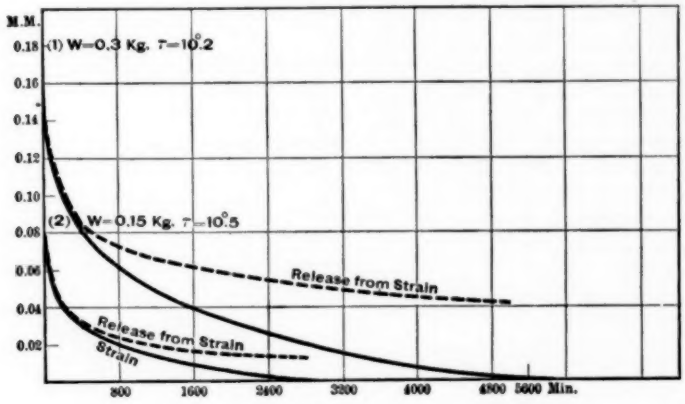


Fig. II.

were only continued for three days. I judge it probable, therefore, that, if there had been no set, the two curves would coincide. *It is, at any rate, certain that the fatigue resulting from long periods of tension disappears in about the same time as that which is required to produce it.*

SILVER WIRE.

Longitudinal Fatigue.

Length	23	m.
Diameter	0.32	mm.
Breaking weight	3.8	kg.
Constant stretching weight	0.65	kg.
Elongation for 1 kg.	38	mm.

Table IX. contains the results of six series of observations. The first four are calculated according to formula 2. The constant α, which is common to all four, is somewhat smaller than in the corresponding cases of brass and copper.

TABLE IX.

SILVER.

$T = 1$ min.

	$W = 0.4$ kg. $L = 1520$ $\tau = 10°$			$W = 0.6$ kg. $L = 2280$ $\tau = 9°.8$			$W = 0.8$ kg. $L = 3040$ $\tau = 9°$			$W = 1$ kg. $L = 3800$ $\tau = 9°.9$		
t	x		Δ	x		Δ	x		Δ	x		Δ
	Obs.	Cal.		Obs.	Cal.		Obs.	Cal.		Obs.	Cal.	
0.5	1.6	1.31	+0.29	2.3	2.08	+0.22	2.7	2.37	+0.33	3.8	3.56	+0.24
1.0	1.1	1.09	+0.01	1.7	1.72	−0.02	2.0	1.97	+0.03	3.0	2.95	+0.05
2.0	0.9	0.90	±0.00	1.4	1.43	−0.03	1.7	1.63	+0.07	2.6	2.45	+0.15
3.0	0.8	0.81	−0.01	1.3	1.28	+0.02	1.5	1.46	+0.04	2.3	2.19	+0.11
5.0	0.7	0.71	−0.01	1.2	1.12	+0.08	1.2	1.28	−0.08	2.0	1.91	+0.09
7.0				1.0	1.02	−0.02	1.1	1.16	−0.06	1.7	1.74	−0.04
10.0	0.6	0.59	+0.01	0.9	0.93	−0.03	1.0	1.06	−0.06	1.5	1.58	−0.08
15.0										1.2	1.42	−0.22
20.0										1.0	1.31	−0.31
40.0												
60.0												
80.0												
	$c = 1.090$ $a = 0.27$			$c = 1.722$ $a = 0.27$			$c = 1.968$ $a = 0.27$			$c = 2.951$ $a = 0.27$		

$x = \frac{c}{t^a}$

$W = 0.6$ kg. $L = 2280$

	$T = 10$ min. $\tau = 9°.0$			$T = 30$ min. $\tau = 7°.8$		
t	x		Δ	x		Δ
	Obs.	Cal.		Obs.	Cal.	
0.5						
1.0	3.4	3.48	−0.08	6.2	6.28	−0.08
2.0	3.0	2.95	+0.05	5.5	5.54	−0.04
3.0	2.7	2.64	+0.06	5.2	5.10	+0.10
5.0	2.2	2.25	−0.05	4.6	4.56	+0.04
7.0	2.0	1.99	+0.01	4.2	4.20	±0.00
10.0	1.8	1.71	+0.09	3.8	3.81	−0.01
15.0	1.4	1.40	±0.00	3.4	3.38	+0.02
20.0	1.2	1.18	+0.02	3.2	3.07	+0.13
40.0	0.6	0.65	−0.05	2.3	2.33	−0.03
60.0				1.9	1.89	+0.01
80.0				1.5	1.58	−0.08
	$c = 3.48$ $p = 0.000776$			$c = 6.28$ $p = 0.001082$		

$x = c - pL \log t.$

The agreement between the observed and calculated values in the first three is exceedingly good, where Δ is, with the exception of the rather uncertain observations for $t=\frac{1}{2}$ min., in every case less than 0.001 min. The fourth series ($W=1$ kg.) is not as good, and I think that perhaps in this case the strain was a little too near the limits of elasticity, where my observations have led me to believe that the fatigue becomes irregular.

TABLE X.

T	x_1		Δ
	Obs.	Cal.	
0.5	1.56	1.71	−0.15
1.0	2.23	2.23	±0.00
3.0	3.72	3.39	+0.33
10.0	4.52	5.35	−0.83
30.0	8.56	8.12	+0.44

The above table gives the values of x_1 for $W=0.6$ kg. and $\tau=20°$ for different values of T. The calculations were made from the formula

$$x_1=2.23\cdot T^{0.38}.$$

x_1 varies nearly proportionally with W, and as a general expression at a temperature of 20° we may write:—

$$x_1=3.74\cdot W\cdot T^{0.38},$$

or in terms of the elongation,

$$x_1=0.00098\,L\cdot T^{0.38}$$

and

$$\lambda=0.00098.$$

The series representing the fatigue for larger values of T are calculated according to the formula

$$x=c-pL\log t,$$

and the agreement is seen to be exceedingly good.

TABLE XI.

SILVER TORSION.

$T = 1$ min.

	$\phi = 60°$ $\tau = 18°.2$			$\phi = 90°$ $\tau = 17°.2$			$\phi = 150°$ $\tau = 20°$		
s	x		Δ	x		Δ	x		Δ
	Obs.	Cal.		Obs.	Cal.		Obs.	Cal.	
0.5	9.5	10.28	−0.78	14.5	14.79	−0.29	28.0	28.05	−0.05
1.0	7.2	7.32	−0.12	10.2	10.53	−0.33	19.8	19.97	−0.17
2.0	5.4	5.21	+0.19	7.8	7.50	+0.30	14.3	14.22	+0.08
3.0	4.4	4.28	+0.12	6.1	6.16	−0.06	11.1	11.67	−0.57
5.0	3.3	3.33	−0.03	4.9	4.79	+0.11	9.4	9.08	+0.32
7.0	2.7	2.82	−0.12	4.2	4.06	+0.14	8.0	7.70	+0.30
10.0	2.3	2.37	−0.07	3.6	3.41	+0.19	6.8	6.46	+0.34
15.0							5.4	5.30	+0.10
20.0							4.6	4.60	±0.00
40.0							3.6	3.28	+0.32
	$c = 7.32$			$c = 10.53$			$c = 12.97$		

$a = 0.49$

$x = \frac{c}{t^a}$

Torsional Fatigue.

The results of three series of observations for $T = 1$ min. are contained in Table XI. The theoretical values are calculated according to formula 2 with the constant $a = 0.49$ common to all the series. x_1 is here nearly proportional to ϕ, and for $\tau = 20°$

$$x_1 = 0.129\,\phi,$$

or by multiplying by 0.014 (the value of one scale division in circular measure), and dividing by ϕ, we get

$$\lambda' = 0.014\frac{x_1}{\phi} = 0.0018,$$

from which

$$\lambda' = 1.84\,\lambda.$$

Kohlrausch[1] found, in his investigation on the torsional fatigue of a silver wire having a diameter of 0.092 mm. for a torsion of 1 min. at a temperature of 20°, $\alpha = 0.39$ and $\lambda' = 0.00102$.

This led me to suspect that perhaps the diameter of the wire might affect the fatigue. I therefore drew out a piece of the same wire used in getting the above results, until its diameter was only about half its former size. For the first few days, probably as a result of the drawing, the fatigue was larger than before, and somewhat irregular, but after a week had passed, I again found the same values for α and λ' as before. However, as my wire was pure silver and Professor Kohlrausch's, as he himself has told me, was not, it is probable that this is the cause of the difference in the results.

The Influence of the Temperature.

The wire for the longitudinal fatigue was not placed in position before the winter set in. On this account I was unable to make any observations on its temperature coefficient.

In Table XII. are given the values of x_1 for four temperatures.

TABLE XII.

τ	x_1 (Torsion)
20°	15.8
16°	14.4
12°	12.3
8°	11.1

$\phi = 120°$ $T = 1$ min.

From this, the value of the temperature coefficient,

$$\gamma = 0.036,$$

which is about the same as for brass and copper.

Fatigue of Long-continued Tension.

Fig. III. shows two curves illustrating the course of the fatigue in silver during strain and after release from strain. For both of these the time of strain was 2880 min., and the stretching weights were respectively 0.4 kg. and 0.2 kg. The results are a little better

[1] Kohlrausch: Pogg. Ann., 1876, Bd. CLVIII., p. 342.

than in brass and copper, but still far from satisfactory. As in the other cases, the silver wire seems to recover from a long-continued fatigue in about the same time as that required to produce it.

In order to allow a more exact comparison than Fig. III. offers in Table XIII., the observed values of x are given for curve No. 1,

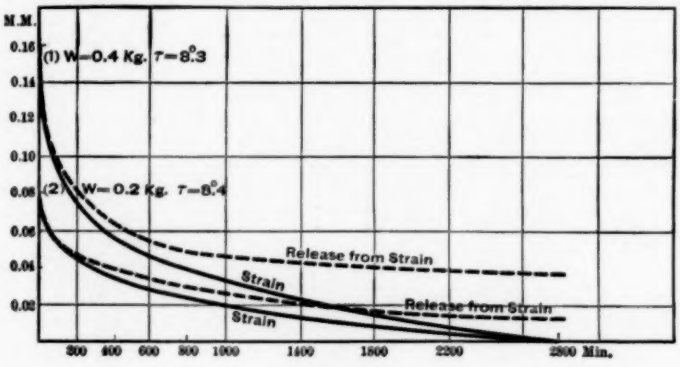


Fig. III.

to which are added the calculated values for the part of the curve showing the fatigue after release from strain. The calculations were made from the formula

$$x = c - pL \log t.$$

The two curves of the fatigue, after release from strain, seem to be fairly similar and nearly proportional to the producing strain, but the similarity and proportionality are both far from perfect. The agreement between the observed and calculated values in No. 1 is also poor.

The Formula, $x = c - pL \log t$.

It was found, on trial, that the formula $x = c - pL \log t$ also represents the course of the fatigue for short periods of strain. Table XIV. shows the first three series of Table IX. calculated according to it.

The agreement between the observed and calculated values seems to be quite as good as in the case where Kohlrausch's

TABLE XIII.

$T = 2880$ min.

$W = 0.4$ kg. $L = 1520$

t	Strain. x Obs.	After Release from Strain. x Obs.	x Cal.	Δ
1	15.0	15.0	15.8	−0.8
2	14.4	14.4	14.8	−0.4
3	14.0	14.0	14.2	−0.2
5	13.6	13.7	13.4	+0.3
10	12.4	13.2	12.4	+0.8
20	11.6	12.0	11.3	+0.7
40	10.6	11.0	10.3	+0.7
60	9.6	10.1	9.7	+0.4
120	8.4	9.3	8.6	+0.7
180	7.4	8.2	8.0	+0.2
300	6.3	6.8	7.3	−0.5
1340	2.4	4.2	5.0	−0.8
2880	0.0	3.4	3.9	−0.5

$\tau = 8°.3$

$c = 15.8$
$p = 0.00227$
$\tau = 8.5$
$x = c - pL \log t$

TABLE XIV.

$T = 1$ min.

t	$W = 0.4$ kg. $L = 1520$ $\tau = 10°$ x Obs.	x Cal.	Δ	$W = 0.6$ kg. $L = 2280$ $\tau = 9°.8$ x Obs.	x Cal.	Δ	$W = 0.8$ kg. $L = 3040$ $\tau = 9°$ x Obs.	x Cal.	Δ
1	1.1	1.07	+0.03	1.7	1.68	+0.02	2.0	1.97	+0.03
2	0.9	0.92	−0.02	1.4	1.45	−0.05	1.7	1.67	+0.03
3	0.8	0.83	−0.03	1.3	1.32	−0.02	1.5	1.49	+0.01
5	0.7	0.72	−0.02	1.2	1.16	+0.04	1.2	1.27	−0.07
7				1.1	1.05	+0.05	1.1	1.12	−0.02
10	0.6	0.57	+0.03	0.9	0.93	−0.03	1.0	0.97	+0.03
	$c = 1.07$			$c = 1.68$			$c = 1.97$		

$p = 0.000329$
$x = c - pL \log t$

formula No. 2 was used. I have also tried to apply my formula to the calculation of the torsional fatigue, but while it represents it fairly well, its accuracy is not as great as Kohlrausch's. Perhaps this last is not to be wondered at, as the molecular motions in torsion are probably more complicated than in stretching.

According to this formula, the change of shape of a body which has been strained is proportional to the log of the time since the strain was removed; then, by differentiation, the velocity of this change is inversely proportional to that time, or

$$-\frac{dx}{dt}=\frac{pL}{t}$$

Summary of the Results.

The longitudinal fatigue in general seems to follow the same laws as the torsional; it is nearly proportional to the deforming strain; the curves, which represent its disappearance, are similar, so long as the time of strain is short; the curve sinks more slowly as this time increases.

The fatigue increases with the temperature, and within the limits observed this increase is proportional to the increase of temperature. The temperature has very little influence on the velocity of disappearance of the fatigue, but apparently for lower temperatures this velocity is slightly increased.

The course of the longitudinal fatigue may be represented by Kohlrausch's formulæ 1 and 2, but it also seems possible to represent it by the more simple formula

$$x=c-pL\log t.$$

In Table XV. are given the values of λ' and λ for a time of strain of 1 min. and a temperature of 20°, and also the temperature coefficients for torsion of all three metals investigated.

The fatigue in copper, silver, and brass are to each other nearly as 7 : 3 : 2 for the torsion, and as 4 : 3 : 2 for the tension. I do not think that the values of λ' and λ in copper can be properly compared, on account of the lack of proportionality between the

fatigue and the deforming strain, and I think it probable that in general the relation between the fatigue of torsion and tension is about as 2 is to 1.

The temperature coefficient both for torsion and tension in all three metals seems to be not far from $\frac{1}{300}$.

The results in regard to the course of the fatigue during long-continued strain, and after release from strain, are unsatisfactory, as all the wires seemed to take a permanent set if the strain lasted more than a few hours. The most that can be said is, that for small strains lasting for several days, the fatigue apparently disappears in about the same length of time as that required to produce it; and, if the strain is very small, the two curves take nearly the same course.

TABLE XV.

	λ' Torsion.	λ Tension.	$\frac{\lambda'}{\lambda}$	γ
Brass	0.0011	0.00059	1.9	0.029
Copper	0.0045	0.0013	3.6	0.034
Silver	0.0018	0.00098	1.8	0.036

From the observations on the silver wire it seems that in general the fatigue is entirely independent of the diameter of the wire.

The poor agreement of the observed and calculated values in the case of the long-continued strain of silver (Table XIII.), together with some other observations made shortly after the wires had been strained beyond the limits of elasticity, has led me to believe that probably the passage of this limit is accompanied by such a disturbance of the molecules of the body that it requires several days before it again assumes its normal condition, and that during this time the fatigue is somewhat irregular.

I hope within a short time to publish further results in regard to the effects of temperature on the fatigue, and also on a possible relation between the elastic limit and the time of strain.

I wish to take this opportunity of thanking Professor Kohlrausch, in whose laboratory this research was carried out, for his great kindness and many valuable suggestions during the progress of the work.

PHYSICAL LABORATORY OF THE UNIVERSITY OF WISCONSIN,
December, 1893.

ON THE CONDITION OF THE ETHER SURROUNDING A MOVING BODY.

BY WILLIAM S. FRANKLIN AND EDWARD L. NICHOLS.

THE problem with which this paper deals is really that of the existence of terms involving products of currents and ordinary velocities in the general expression for the kinetic energy of an electric current. It is our purpose to describe an experiment bearing upon this question, and to give the theory.

I.

Introductory. — The only rapid motion which is experimentally attainable is rotary motion. Such motion is cyclic[1] when the rotating body possesses circular symmetry with respect to the axis of rotation.

The expression for the kinetic energy of a system in cyclic motion[2] does not involve the co-ordinate of which the given velocity is the rate, nor the rates, when they change, of the geometric parameters. If a rotating body is not symmetrical, the motion is not strictly cyclic, for the energy of the system will depend upon the value of the cyclic variable if the non-symmetrical rotating body produces waves of any kind in the surrounding medium. If no waves are produced, then the rotation of a non-symmetrical body may be treated as cyclic.[3]

In this paper an hypothetical motion of the ether in the neighborhood of a moving body is assumed as the basis for the exist-

[1] See von Helmholtz, "Principien der Statik monocyklischer Systeme," Crelle's Journal, Vol. 97, p. 111 and p. 317.

[2] The characteristic of cyclic motion is that it can be completely specified by a number of constant or slowly varying geometric parameters and a single constant, or slowly varying, generalized velocity.

[3] See Boltzmann, Vorlesungen über Maxwell's Theorie der Electricität und des Lichtes, Erster Theil.

ence of kinetic energy terms involving products of velocity and current, so that it will not be allowable for our purpose to consider the rotation of a non-symmetrical body as cyclic. The theoretical and experimental parts of the paper will be limited to the case of circularly symmetrical coils rotating about their axes of symmetry. Electric circuits moving otherwise than cyclically may present no phenomena essentially different from the phenomena due to cyclic motion, except in the production of electro-magnetic waves due to the moving current, and perhaps ether-waves of some kind due to the moving matter. Kinetic energy terms involving products of current and velocity have nothing to do with this production of waves, so that the study of cyclic motion of circuits may lead to results of more general significance than might at first be anticipated, considering the restricted nature of cyclic motion.

II.

Monocyclic System. — Let $x_1, x_2, x_3, \cdots$ be geometric parameters which determine the character of a monocyclic motion; let y_1 be the cyclic variable, and $\dot{y}_1$ its rate. Let v be the velocity of the moving substance at the point p; v is a vector function of the position of the point p, involving the x's and $\dot{y}_1$, and it is linear[1] with respect to y_1; so that we may write

$$v=\dot{y}_1 fi+\dot{y}_1 gj+\dot{y}_1 hk, \qquad (1)$$

where ijk are rectangular unit vectors, and f, g, and h are determinate scalar functions of the x's and of the co-ordinates of the point p. The form of f, g, and h depends upon the nature of the cyclic motion. If the moving mechanism has finite rigid parts, f, g, and h will be discontinuous functions. In moving matter, f, g, and h may be two-valued, one value referring to the moving matter, the other to the ether which may be carried with moving matter at a distinct velocity.[2]

Equation (1) and all that follow apply to any monocyclic or bicyclic systems whatever. We shall, however, keep in mind only motions of rotation, including any possible associated motion of

[1] See von Helmholtz, *loc. cit.*, pp. 319, 320.

[2] See J. J. Thomson, Recent Researches in Electricity and Magnetism, p. 544.

the ether, and the cyclic motion which constitutes the electric current.

The total kinetic energy of the monocyclic system is

$$\tfrac{1}{2}L_1\dot{y}_1^2=\frac{\dot{y}_1^2}{2}\iiint(f^2+g^2+h^2)\rho dxdydz, \qquad (2)$$

in which ρ is the density of the moving substance. The integration is to be extended throughout all space, or where v is other than zero. If v is other than a linear velocity, then ρ cannot be so simply defined. If v is an angular velocity, and if the energy due to it is finite, then v must be at each point of the same order of magnitude as the reciprocal of the linear dimensions of the ultimate rotating parts of the moving substance, and ρ will be the moment of inertia of one of these parts multiplied by the number of them in unit volume. See IV. *b*.

The quantity

$$L_1=\iiint(f^2+g^2+h^2)\rho dxdydz \qquad (3)$$

may be called the inertia of the monocyclic system.

In case $\dot{y}_1$ is an electric current, then $(f^2+g^2+h^2)\rho$ becomes $(\alpha^2+\beta^2+\gamma^2)\frac{\mu}{4\pi}$, where α, β, and γ are the components of the magnetic field at p, and μ is the permeability of the medium. We cannot yet separate $(\alpha^2+\beta^2+\gamma^2)\frac{\mu}{4\pi}$ into its two factors v^2 and ρ; when we can do this we shall be able to specify strength of magnetic field as a velocity and permeability as some kind of moment of inertia per unit volume. Equation (3) leads to an expression for the coefficient of self-induction in a circuit as $\frac{1}{4\pi}$ into the volume integral of the square at each point of the magnetic field per unit current in the coil.

III.

Bicyclic System. — Let y_1 and y_2 be the cyclic variables concerned in a bicyclic motion, and $\dot{y}_1$ and $\dot{y}_2$ their respective rates. Let $x_1, x_2, x_3, \cdots$ and $z_1, z_2, z_3, \cdots$ be the geometric parameters which

determine the character of the respective cycles. The velocity of the moving substance at the point p will now, in general, be the superposition of two vector parts linear with respect $\dot{y}_1$ and $\dot{y}_2$ to respectively. Let these parts be $\dot{y}_1 v_1$ and $\dot{y}_2 v_2$. As in equation (1) we have

$$\dot{y}_1 v_1 = \dot{y}_1 f_1 i + \dot{y}_1 g_1 j + \dot{y}_1 h_1 k.$$

$$\dot{y}_2 v_2 = \dot{y}_2 f_2 i + \dot{y}_2 g_2 j + \dot{y}_2 h_2 k. \tag{4}$$

The total velocity at the point p is $\dot{y}_1 v_1 + \dot{y}_2 v_2$. There is nothing inconsistent in thus indicating the sum of the vectors $y_1 v_1$ and $y_2 v_2$, even if they are velocities of different kinds, and for the moment we shall consider them to be alike in kind so that the energy per unit volume at the point p is $\frac{1}{2}(\dot{y}_1 v_1 + \dot{y}_2 v_2)^2 \rho$, or $\frac{1}{2}\dot{y}_1^2 v_1^2 \rho + \dot{y}_1 \dot{y}_2 \rho S \cdot v_1 v_2 + \frac{1}{2}\dot{y}_2^2 v_2^2 \rho$, and the total kinetic energy of the system embraces three terms which are the volume integrals of the respective terms in this expression; that is,

$$W = \tfrac{1}{2} L_1 \dot{y}_1^2 + M \dot{y}_1 \dot{y}_2 + \tfrac{1}{2} L_2 \dot{y}_2^2, \tag{5}$$

where

$$L_1 = \iiint (f_1^2 + g_1^2 + h_1^2) \rho dx dy dz = \iiint v_1^2 \rho dx dy dz,$$

$$L_2 = \iiint (f_2^2 + g_2^2 + h_2^2) \rho dx dy dz = \iiint v_2^2 \rho dx dy dz, \tag{6}$$

$$M = \iiint (f_1 f_2 + g_1 g_2 + h_1 h_2) \rho dx dy dz = \iiint S v_1 v_2 \cdot \rho dx dy dz. \tag{7}$$

We shall call the quantity M the *concurrence* of the two cyclic motions, since it is a generalization of that which for simple vectors involves the cosine of an included angle. M is evidently a function of the x's, of the z's, and of the co-ordinates which determine the relative positions of the cycles. When M is a maximum, the two cycles are said to be *concurrent*. If v_1 and v_2 are everywhere parallel, $V \cdot v_1 v_2 = 0$, and the two cycles are said to be *identically* concurrent. If $M = 0$, the two cycles are said to be *orthogonal*; if $S \cdot v_1 v_2 = 0$ everywhere, that is, if v_1 and v_2 are everywhere mutually perpendicular, then the two cycles are said to be *identically* orthogonal. It is worthy of note that a cycle has in a general way many *vector* properties which depend upon a number of independent numerical specifications. If $\dot{y}_1 v_1$ and

$\dot{y}_2 v_2$ are velocities of different kinds, angular and linear velocities, for example, then the quantity M may have no physical significance. Equation (7) will, however, be retained as its mathematical definition; ρ being merely some scalar function of position. The reason for thus retaining a conception of M is that we do not wish to assume that no part of the energy of a current in cyclic motion can depend upon this motion when the motion produced in the ether by the moving coil is of a different kind from the motion produced in the ether by a current in the coil. (See VI.) Equation (7) is easily interpreted for the case of two electric circuits. It leads to an expression for the coefficient of mutual induction of two coils as $\frac{1}{4\pi}$ into the volume integral of the scalar product at each point of the magnetic fields per unit current in the respective coils.

IV.

The Motion of the Ether near Moving Bodies. — In the expression for the kinetic energy of a system it is scarcely conceivable that terms involving the product of generalized velocities can occur unless parts of the moving system depend for their actual motion upon both these generalized velocities.[1] Hence we must consider the possible motion of the ether in the neighborhood of moving bodies as the basis for the existence of mutual energy terms in a bicycle consisting of a rotating coil.

We shall consider two possibilities in the motion of the ether: —

(*a*) That the ether in the neighborhood of a moving body moves as a viscous fluid.[2]

(*b*) That the ether may be cellular in structure, adjacent cells being positive and negative respectively.[3] Such an ether might be capable of moving as a viscous fluid, as in (*a*), without the finite angular velocity, due to the curl, if it exists, of the linear velocity, contributing sensibly to the energy of the motion (see II.). Such

[1] See Maxwell's Treatise, Vol. II., p. 199, 2d edition.

[2] See J. J. Thomson, Recent Researches in Electricity and Magnetism, p. 543.

[3] See Oliver Lodge, Modern Views on Electricity.

an ether would also be capable of a violent rotary motion of its ultimate parts or cells, independently of any linear velocity, and without breaking down the connecting mechanism. If moving matter should have a stronger hold upon one set of these cells than upon the other, this violent rotary motion might be set up in the neighborhood.

The lines of flow of a viscous fluid in the neighborhood of a rotating body are circles coaxial with the rotating body. Hence a bicycle consisting of a current in a rotating circular coil would be identically orthogonal if the ether move as a viscous fluid. A bicycle consisting of a current in a rotating uniformly wound Gramme ring would be identically concurrent if the ether move as a viscous fluid.

The vortex lines in the ether, moving as described in (*b*), in the neighborhood of a long rotating cylinder would be straight lines parallel to the axis. For a short cylinder the vortex lines would approximate to the magnetic lines due to a current in a circular coil. Hence a bicycle consisting of a current in a rotating circular coil would be concurrent if the ether move as described in (*b*). A bicycle consisting of a current in a rotating Gramme ring would be identically orthogonal for this case of ether motion.

If the quantity M be found to have physical existence for a rotating circular coil, the case (*b*) of ether motion would be, to some extent, verified. If the quantity M be found to have physical existence for a rotating Gramme ring, the case (*a*) of ether motion would be verified. If M exists for both coils, then the ether could perhaps be considered to move in both ways in the neighborhood of a moving body.

V.

The phenomena which depend upon the mutual action of two cycles during a permanent or varying state of the bicyclic motion.

(A) *Permanent State.* — Let $X \ldots$ and $Z \ldots$ represent the forces which tend to change the respective x's and z's, or any of the co-ordinates which specify the relative position of the cycles, in a bicyclic motion. ($X \ldots$ and $Z \ldots$ being understood to mean only

those portions of these forces which depend upon the mutual action of the cycles.) Then

$$X = -\frac{dM}{dx}\dot{y}_1\dot{y}_2,$$

$$Z = -\frac{dM}{dz}\dot{y}_1\dot{y}_2, \text{ etc.,} \qquad (8)$$

and from Lagrange's equation, total force $X = \frac{d}{dt}\frac{dW}{d\dot{x}} - \frac{dW}{dx}$; in which W is the total kinetic energy (see equation (5)), and is independent of $\dot{x}$, which is, in fact, equal to zero. The force due to mutual action depends only upon the mutual energy term.

(B) *Varying State.* — When the state of a cyclic motion is changing, it is not strictly cyclic. In this case the expression (1) for the velocity of the moving substance at each point fails. Indeed, equation (1) is in many respects the most evident characteristic of a cyclic motion. If the change of state takes place slowly, the motion may be treated as cyclic. The requisite slowness of change depends upon the quickness with which any change of state is established throughout the moving mechanism, and upon the degree of approximation required in equation (1). A rotating rigid body may change its angular velocity with great rapidity without losing the character of cyclic motion. If the ether is involved in such motion, then the angular velocity must not change so rapidly that the ether motion is not sensibly conformable at each instant to the motion of the body. An electric current, however, may change with great rapidity without becoming essentially non-cyclic in character; for the state of motion of the ether in the neighborhood of a current is propagated with the velocity of light, and the greater part of the energy is in the immediate neighborhood of the coil. In the ordinary treatment of alternating currents, the kinetic energy of the current is assumed proportional to the square of the current at each instant, which by equation (2), a result of (1), amounts to assuming the current to be sensibly a cyclic motion at each instant.

Let Y_1 and Y_2 be the forces, due to mutual action of the cycles, and depending upon a changing state, which tend to

change the cyclic rates $\dot{y}_1$ and $\dot{y}_2$ respectively; then by Lagrange's equation,

$$Y_1 = M\frac{d\dot{y}_2}{dt}, \tag{9}$$

$$Y_2 = M\frac{d\dot{y}_1}{dt}. \tag{10}$$

The impulses of these forces, Y_1 and Y_2, for any integral changes, $\dot{y}_1$ and $\dot{y}_2$, of these cyclic rates, are:

$$\text{Impulse of } Y_1 = M\dot{y}_2,$$
$$\text{Impulse of } Y_2 = M\dot{y}_1. \tag{11}$$

There are then three methods depending upon the mutual action of two cycles by which the mutual energy term of a bicycle consisting of a rotating coil may be determined. (*a*) By observing the effects of the forces X or Z. This requires the observation of the effect of a force upon a rapidly moving rigid body. [The experiment described by Maxwell on page 203 of the second volume of his Treatise is an attempt to detect one of these forces, X, since the angle θ in his discussion is one of the parameters of the motion, which, inasmuch as he has ignored electro-magnetic and other ether-waves, may be considered to be cyclic.] (*b*) By observing the effect of the impulsive torque $M\dot{y}_2$, due to the establishment of a current $\dot{y}_2$ in the coil.[1] (*c*) By observing the effects of the impulsive electromotive force $M\dot{y}_1$, due to a change, $\dot{y}_1$, in the angular velocity of the coil.

Let a be the observed angular velocity produced in a coil by the establishment of a current $\dot{y}_2$, and let K be the moment of inertia of the coil; then

$$Ka = M\dot{y}_2. \tag{12}$$

This equation will enable us to calculate the value of M when K, a, and $\dot{y}_2$ are known, or to assign an upper limit to M when we have estimated from the conditions of an experiment the least observable angular velocity, and have failed to detect any angular velocity when the current is changed.

[1] See Maxwell's Treatise, Vol. II., p. 201.

Let k be the reduction factor of a ballistic galvanometer, r the combined resistance of the galvanometer and rotating coil, and d the deflection produced by the impulsive electromotive force $M\dot{y}_1$;

then
$$\frac{M\dot{y}_1}{r} = kd, \tag{13}$$

from which inferences, similar to those deduced from equation 12, may be drawn, provided that d has been observed; or, in case of no deflection, when the least observable d has been estimated.

VI.

Another Means for the Detection of the Quantity M. — If it be assumed that M must depend not only upon motion at each point due to both cycles, but also that these two motions must be alike in kind (see III.), then the existence of the term $M\dot{y}_1\dot{y}_2$ would require the disturbance of the ether in the neighborhood of a moving body to be, partly at least, of the nature of magnetic field, so that a suspended magnet would be affected by the motion. In this case, the numerical value of M cannot be rigorously calculated from an observed deflection, nor can the upper limit be easily assigned. A roughly approximate, and perhaps questionable, reduction formula may be derived as follows: assume the motion and current to be identically concurrent, so that v_1 and v_2 are everywhere parallel, and assume the ratio $\frac{v_1}{v_2} = a$ to have everywhere the same value. Substitute the value of $v_1 = av_2$ in equation (7); then

$$M = \int\int\int S \cdot v_1 v_2 \rho dx dy dz = a \int\int\int v_2^2 \rho dx dy dz = aL_2. \tag{14}$$

Let d_1 be the observed deflection produced upon the suspended magnet by an angular velocity $\dot{y}_1$ of the coil, and let d_2 be the observed deflection produced by a known current $\dot{y}_2$ in the coil.

Then
$$a = \frac{d_1\dot{y}_2}{d_2\dot{y}_1}, \tag{15}$$

from which expression, when the coefficient of self-induction, L_2, of the coil is known, $M = aL_2$ (14) gives the value of M.

The existence of the quantity M, depending as it does upon motion produced in the ether by a rotating body, requires part of the energy of any rotation to be due to the motion of the ether. It is interesting to notice that the amount of this energy may be estimated when $\dot{y}_1 v_1$ and $\dot{y}_2 v_2$ are of the same nature. Let the coil rotate at angular velocity $\dot{y}_1$, producing at each point a magnetic field $\dot{y}_1 v_1$, which is equal to $\dot{y}_1 a v_2$. Then the field $\dot{y}_1 a v_2$ is that which would be produced by a current $\dot{y}_2 = \dot{y}_1 a$ in the coil of which the kinetic energy is $\frac{1}{2} L_2 \dot{y}_1^2 a^2$. Since $\frac{1}{2} K \dot{y}_1^2$ is the total energy of the rotation, the ratio of energy of the motion of the ether to the total energy is

$$\frac{L_2 a^2}{K}. \qquad (16)$$

VII.

Apparatus and Experimental Results. — The apparatus used was designed for carrying out method (*c*) of V. and the method described in VI.; the method (*c*) of V. being applied to both coils A and B (see below), and the method of VI. being applied only to coil A.

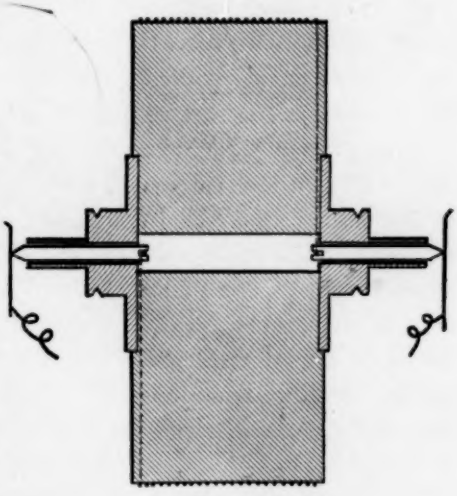


Fig. 1.

Two coils, A and B, were constructed, each one foot in diameter and five inches thick. They were wound upon cores of well-seasoned pine (Fig. 1), with hollow brass trunnions securely fastened to the core by means of brass discs five inches in diameter. The hubs of these trunnions were about two inches in diameter, and were grooved to receive a driving cord. The total weight of each coil was about 15 pounds. Coil A consisted of 61 turns of No. 18 B. & S.

bare, hard-drawn, copper wire, laid in a thread turned upon the periphery of the core. It had a resistance of about 1.3 ohms.

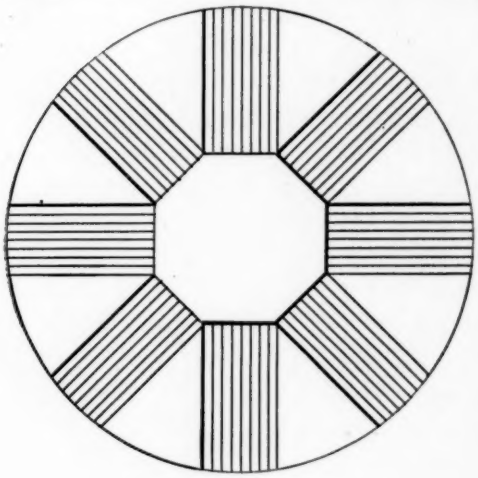


Fig. 2.

Coil *B* (Figs. 2 and 3) consisted of a Gramme ring wound in eight sections of 36 turns each, of double cotton-covered No. 20 B. & S. copper wire, connected continuously and bound with a band of brass wire. It was necessary to wind coil *B* thus in sections, leaving spaces on each side in which wooden lugs were placed, forming faces against which circular boards of ash, each one foot in diameter, were screwed, giving smooth surfaces against which a brake could be applied. Fig. 2 shows the hollow wooden drum upon which coil *B* was wound. The resistance of *B* was 3.2 ohms. The coils were carried on hard-wood bearings, in a heavy wooden frame. Their terminals were connected, without solder, to insulated copper rods held in the hollow trunnions and ending in carefully centered points, against which pressed copper springs (see Figs. 1 and 3). The brake, which was supported in the same frame, consisted of a pair of hinged wooden jaws, which could be pressed with equal force against the two ends

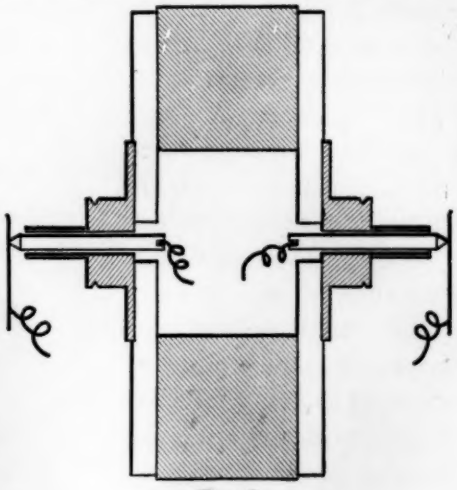


Fig. 3.

of the rotating cylindrical coil by means of a system of cords leading to a strong pedal. The rubbing surfaces of the brake consisted of two leather-covered blocks, securely fastened to each of the jaws so as to rub at points diametrically opposite. With this arrangement the bearings were not subject to any great end or side thrust during the application of the brake. The brake was thoroughly effective, bringing either coil from a speed of 60 revolutions per second to a dead stop in about a second or a second and a half.

The galvanometer used was constructed in the shop of the physical laboratory at Ames, Iowa. The elements of the astatic pair contained four magnets each. They were very nearly equal in strength, and were only two inches apart, — the mirror being placed below the coils instead of between them. By this arrangement the galvanometer was rendered comparatively insensible to magnetic disturbances. The galvanometer was provided with freshly made magnets just before being used, — a precaution which should always be taken in preparing for any important work requiring the last degree of sensitiveness, and care was taken to send no currents of any ordinary strength through the instrument. In the preparation of the magnets the following precautions were taken. Piano wire $\frac{1}{4}$ mm. in diameter was straightened by subjecting it to slight tension at a low red heat, and was cut into two-inch lengths. These were placed, two or three at a time, in an acute V-shaped iron trough, and after being heated uniformly to a cherry-red heat (800° C.) in a Bunsen flame, were quickly dropped into cool water. Two small pieces of exactly the same length were then cut from the central portion of each, and magnetized under similar conditions. The cutting was done by placing the hardened wire upon a smooth block of hard wood, and pressing an edged tool against it. If this procedure be carefully followed a highly astatic pair may always be obtained.

The two small magnets thus made from each piece were used, one in each of the elements of the astatic system. The galvanometer had a resistance of 15 ohms with its coils in series. When so arranged, and with a half-period of seven seconds, and scale distance of 120 cm., a deflection of 1 mm. corresponded to

6×10^{-10} amperes. During the course of the experiments the coils of the galvanometer were connected in multiple so as to have a total resistance of about one ohm.

It was found necessary to make the circuit external to the galvanometer, which was housed in a box, entirely of copper, and without soldered joints. The key for opening and closing the circuit was made by scraping the ends of the copper wires and bending them into hooks, one of which was fastened to a small weight, and the other to the edge of the table. Even with such precautions it was found necessary to include in the circuit about three feet of a heavy copper rod, No. 0000 B. & S., through which a current from a gravity cell could be passed, so as to balance any existing thermo-electromotive force.

The suspended needle for carrying out the method of VI. consisted of a widely separated astatic pair, each element consisting of two magnets about 6 mm. long, and $\frac{1}{3}$ mm. in diameter, carrying a small mirror, and suspended in a brass tube which was supported above the coil by a scaffold fastened to the brick walls of the room.

The coils were driven by a cord from a small electric motor, placed 75 feet away from the coil. The galvanometer was conveniently near the coil. Coil *B* was driven, during a trial run, at a speed which approached 100 revolutions per second. While taking observations, however, the speed was always approximately 60 revolutions per second.

The extreme sensitiveness of such a low-resistance galvanometer to electromotive force is very striking. The movement of an oil can, having an iron snout, used in oiling the bearings of the coil, for example, would produce a quick motion of the galvanometer needle through several centimeters of deflection. One of the connecting wires, about eight feet long, happened to be stretched horizontally east and west. It performed about twelve half-vibrations per second when disturbed. The electromotive force induced in this wire by the earth's field while vibrating through an amplitude of about an inch was found to force a synchronous vibration upon the galvanometer needle with an amplitude of about a centimeter on the scale, the proper half-period of the

galvanometer needle being about seven seconds. Placing the finger on one side of the hooked junction of the key gave a deflection of three centimeters; placing the finger on the back of one of the commutator springs gave 6 cm. deflection. During some preliminary experiments, when the wires leading to the galvanometer were soldered to the ends of these springs, the effect of touching the finger for a second or two to the other end of the spring was to throw the galvanometer off the scale, although the springs were thin, and about four inches long. The copper rod shunt, already described, was used principally to balance electromotive forces produced by the warming of the bearings. Obscure magnetic disturbances in the neighborhood of the coil — especially when coil *A* was in circuit — gave troublesome irregular movements of the galvanometer. With this coil, *A*, it was found that there was enough undetachable iron about the person of the assistant manipulating the brake, even when he wore slippers containing no iron nails, to produce a throw on the galvanometer for every movement of his body. This was shown to be due to inductive action on the coil, since it did not occur with open circuit. It was found that the movement necessary in applying the brake produced from two to five millimeters of throw, the direction of which was always the same, but the magnitude of which could not be entirely controlled, even when care was taken to perform the movement always in the same way. The motion of the assistant in walking up to the coil after starting the motor (or after going through the motions of starting the motor) produced always, with coil *A*, about 20 mm. deflection, "making ready" for applying the brake, 2 mm. The signal "now," for the application of brake, was followed by 2 to 5 mm. deflection, when the assistant went through the motions of braking with the coil stationary. With coil *B* no serious trouble of this kind was experienced. The effect of the magnetism of the motor — the field being continually excited — was slightly noticeable with coil *A*, and was eliminated by reversing the connections of the motor after several runs. This was perhaps a needless precaution, as the coil was driven alternately in opposite directions by crossing the driving cord. While the coils were in motion, the galvanometer was gen-

erally unsteady, and it was always necessary to await a favorable moment before giving the signal for brakes.

The result of thirty runs with coil A, of which naturally a few of the later ones were more reliable than all the others, failed to show any distinct deflection due to the stopping of the coil. Under the conditions of the experiment a deflection of 2 mm. due to this cause could certainly have been detected, and even 1 mm. seems a reasonable upper limit. The galvanometer during these observations gave 1 mm. throw for 8 c.g.s. units of electromotive impulse with the coil in circuit. Hence kr in equation (13) has a numerical value 8. $\dot{y}_1$ was equal to 375 radians per second; so that

$$M < 0.021 \text{ c.g.s. for coil } A.$$

The extreme smallness of this number is best exhibited by comparing it with the values of L_1 and L_2. L_1 is the moment of inertia of the rotating body. As it seems reasonable to ascribe as much importance to the moving wood as to the moving wire of the coil, its value is $L_1 = 700000$ and $L_2 = 1500000$ (L_2 being the coefficient of self-induction of the coil). With the above limiting value for M, it is found that $M\dot{y}_1\dot{y}_2$ is less than $\frac{1}{66000000}$ of the total energy of the bicyclic system when $\dot{y}_1$ and $\dot{y}_2$ are numerically equal.

With coil B the impulsive electromotive force produced by stopping was certainly not large enough to give a deflection of $\frac{1}{2}$ mm. The sensitiveness of the galvanometer to impulsive electromotive force was somewhat less with coil B in circuit than with coil A, on account of the increased resistance; but 10 c.g.s. units of electromotive impulse gave 1 mm. deflection, so that kr in equation (13) is equal to 10. The value of $\dot{y}_1$ being 375 as before, we find

$$M < 0.013,$$

for coil B, $L_1 = 700000$ g.cm.^{2} as before, and $L_2 = 1000000$ (approximately); so that when $\dot{y}_1$ and $\dot{y}_2$ are equal, $M\dot{y}_1\dot{y}_2$ is less than $\frac{1}{60000000}$ of the total energy of the system.

The method of VI. was applied to coil A with the following results: The suspended needle itself was quite steady during the motion, the greatest trouble arising from an unconquerable

vibratory motion of the telescope stand, which was tall and unstable. It was, however, certain that d_1 in equation (15) was less than 1 mm., the speed being 375 radians per second. A current $\frac{1}{90000}$ c.g.s. in A gave a deflection of 16 mm. $= d_2$, whence from (15)

$$a < \frac{1}{64000000000}.$$

Since $L_2 = 1500000$, equation (14) gives $M < 0.003$. The expression (16) has the approximate value 10^{-17}, so that if the assumptions of VI. hold true, less than one hundred thousand million millionth part of the energy of the rotating dead coil depends upon the motion in the ether produced by its rotation.

ITHACA, N.Y., January, 1894.

THREE PROBLEMS IN FORCED VIBRATION.

BY WILLIAM S. FRANKLIN.

THE following is a discussion of three problems in forced vibration. The third problem is simple and well known, and as Helmholtz has said, it alone completely answers the problem of forced vibration. It is given here for the sake of the graphical solution which, though certainly not new, seems never to have been mentioned in this connection.

PROBLEM I. — *An unknown periodic force F acts upon a body, the motion of which is observed. It is required to find the harmonic components of F.*

We have

$$F=\sum_m A_m \operatorname{Cos} mqt+\sum_m B_m \operatorname{Sin} mqt, \tag{1}$$

in which the A's and B's are the quantities to be determined.

Let x be the variable co-ordinate of the body; then from the observed motion we have

$$x=\sum_m C_m \operatorname{Cos} mq(t+\delta)+\sum_m D_m \operatorname{Sin} mq(t+\delta), \tag{2}$$

in which everything is known except δ, a time interval introduced in order that t may be the same in equations (1) and (2). To determine the A's and B's, the motion due to F must be calculated and compared with the observed motion. The total motion due to F is the superposition of the various simply harmonic motions due to the respective terms of (1) separately.

Consider the equation (compare equation (25))

$$\frac{d^2x}{dt^2}+\beta\frac{dx}{dt}+\gamma x-Ae^{imqt}=0, \tag{3}$$

in which β is the coefficient of damping; $\gamma=\frac{4\pi^2}{\tau^2}$, where τ is the proper period of vibration of the body, the mass of which is taken

as unity for simplicity; x is the displacement, complex of course; and $i=\sqrt{-1}$. Equation (3) can be broken up into two ordinary equations, viz.:

$$\frac{d^2x}{dt^2}+\beta\frac{dx}{dt}+\gamma x-A\,\mathrm{Sin}\,mqt=0, \tag{4}$$

in which ix is understood to be the imaginary part of x in (3), the i being cancelled; and

$$\frac{d^2x}{dt^2}+\beta\frac{dx}{dt}+\gamma x-A\,\mathrm{Cos}\,mqt=0, \tag{5}$$

in which x is understood to be the real part of x in (3). Hence if we obtain the solution of (3), the real part of this solution will be the solution of (5); *i.e.* the motion due to the force $A\,\mathrm{Cos}\,mqt$. The imaginary part of this solution, dropping i, will be the motion due to the force $A\,\mathrm{Sin}\,mqt$. The solution of (3), for our purpose, is

$$x=Be^{imqt}, \tag{6}$$

in which B is unknown. Substituting x, $\frac{dx}{dt}$, and $\frac{d^2x}{dt^2}$ from (6) in (3), we obtain an equation, free from t, from which B may be determined. Substituting this value of B in (6), and breaking up the resulting equation into its real and imaginary parts, we have, writing A_m and B_m in turn for A,

$$x=\frac{A_m(\gamma-m^2q^2)}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}\mathrm{Cos}\,mqt+\frac{A_m mq\beta}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}\mathrm{Sin}\,mqt.$$

This expression gives the simple harmonic motion maintained by the force $A_m\,\mathrm{Cos}\,mqt$; and

$$x=\frac{B_m(\gamma-m^2q^2)}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}\mathrm{Sin}\,mqt-\frac{B_m mq\beta}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}\mathrm{Cos}\,mqt$$

gives the simply harmonic motion maintained by the force $B_m\,\mathrm{Sin}\,mqt$. The total motion maintained by F is then

$$x=\sum_m\left[\frac{A_m(\gamma-m^2q^2)}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}-\frac{B_m mq\beta}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}\right]\mathrm{Cos}\,mqt$$

$$+\sum_m\left[\frac{A_m mq\beta}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}+\frac{B_m(\gamma-m^2q^2)}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}\right]\mathrm{Sin}\,mqt. \tag{7}$$

Expanding $\text{Cos}\, mq(t+\delta)$ and $\text{Sin}\, mq(t+\delta)$ in equation (2), we have for the observed motion

$$x = \sum_m [C_m \text{Cos}\, mq\delta + D_m \text{Sin}\, mq\delta] \text{Cos}\, mqt$$
$$+ \sum_m [D_m \text{Cos}\, mq\delta - C_m \text{Sin}\, mq\delta] \text{Sin}\, mqt. \qquad (8)$$

Since the calculated and observed motions are identical, all assumed conditions being realized, we obtain, by equating the coefficients of $\text{Sin}\, mqt$ and $\text{Cos}\, mqt$ respectively in (7) and (8), a set of equations, the number of which is equal to twice the largest value of m. From these equations, γ and β being known, δ may be eliminated, and all the A's and B's calculated in terms of one of them.

The ratio of the amplitude of the mth harmonic of the impressed force to the amplitude of the mth harmonic of the motion approaches zero as m increases. When m is large so that equation (2) must be considered as a finite series, small errors in the determination of the motion introduce very large errors into the values of A_m and B_m. Similar remarks apply to problem II. in regard to the summation with respect to m.

If the motion due to the force F be observed before it becomes steady, then in addition to the above motion there will be a superposed vibratory motion of the body with its *proper* period and damping, such that at the instant at which F begins to act, the velocity and displacement pertaining to this proper motion will be equal and opposite to the velocity and displacement pertaining to the maintained vibrations each to each. In that case also the problem is easily solved.

The motion produced by a periodic force is *similar* to the force when corresponding harmonics of the motion and of the force have their amplitudes in a constant ratio and their phase identical. By comparing (1) and (7), this is seen to require that

$$\frac{\gamma - m^2q^2}{(\gamma - m^2q^2)^2 + m^2q^2\beta^2}$$

be independent of m, and that

$$\frac{mq\beta}{(\gamma - m^2q^2)^2 + m^2q^2\beta^2}$$

be equal to zero. These two conditions are satisfied if γ is very large, *i.e.* if the proper period of the body is very short. Slight differences in the phase of corresponding harmonics have less effect than differences in amplitude in producing dissimilarity. It follows, therefore, that for a given large value of γ the similarity may be made more nearly exact by so choosing $\beta(=\sqrt{2\gamma})$ that the middle term in $(\gamma-m^2q^2)^2$ is annulled by $m^2q^2\beta^2$. This is easily seen to be true by arranging (7) thus:

$$x=\sum_m[X\,\mathrm{Cos}\,(mqt-\theta)+Y\,\mathrm{Sin}\,(mqt-\theta)],$$

in which

$$X=\frac{A_m}{\sqrt{(\gamma-m^2q^2)^2+m^2q^2\beta^2}},\qquad Y=\frac{B_m}{\sqrt{(\gamma-m^2q^2)^2+m^2q^2\beta^2}},$$

and

$$\mathrm{Sin}\,\theta=\frac{mq\beta}{\sqrt{(\gamma-m^2q^2)^2+m^2q^2\beta^2}}.$$

The point in question is to make X and Y independent of m with little reference to the value assumed by θ. However, $\beta\frac{dx}{dt}$ (see equation (3)), is ordinarily a roughly approximate expression for damping force, so that to choose β large may seriously complicate the motion. It is best to select a small value for β in any case.

PROBLEM II. — *An alternating current flows in a wire stretched between the poles of a constant electro-magnet. The motion of a point of the wire is observed; required the harmonic components of the current.*

Let

$$f=\sum_n A_n\,\mathrm{Sin}\,npx \qquad (9)$$

be the magnetic field at the point x of the wire, in which the A's are known, $p=\frac{2\pi}{l}$, and l is the length of the wire. The origin is to be taken at one end of the wire, the undisturbed wire being chosen as X-axis, and f being everywhere parallel to the Z-axis. Let

$$I=\sum_m[B_m\,\mathrm{Sin}\,mqt+C_m\,\mathrm{Cos}\,mqt] \qquad (10)$$

be the alternating current, in which $q=\frac{2\pi}{\tau}$, τ being the period of the alternations. The B's and C's are required.

The force F per unit length per unit mass of wire is proportional to fI. Ignoring the proportionality factor, we have

$$F=\sum_m\sum_n A_nB_m \operatorname{Sin} npx \operatorname{Sin} mqt+\sum_m\sum_n A_nC_m \operatorname{Sin} npx \operatorname{Cos} mqt. \quad (11)$$

The total motion maintained by F is obtained by the superposition of the motions maintained by the separate terms. These separate motions are to be found by solving the equations

$$\frac{d^2y}{dt^2}+\beta\frac{dy}{dt}-c^2\frac{d^2y}{dx^2}-A_nB_m \operatorname{Sin} npx \operatorname{Sin} mqt=0, \quad (12)$$

and

$$\frac{d^2y}{dt^2}+\beta\frac{dy}{dt}-c^2\frac{d^2y}{dx^2}-A_nC_m \operatorname{Sin} npx \operatorname{Cos} mqt=0, \quad (13)$$

in which β is the damping coefficient of the wire per unit length, and c^2 is a constant depending upon the tension and mass of wire.

These equations do not admit of any great simplification by the introduction of complex quantities as in problem I.

The maintained vibrations depending upon (12) are of form[1]

$$y=D_{mn} \operatorname{Sin} npx \operatorname{Sin} mqt+E_{mn} \operatorname{Sin} npx \operatorname{Cos} mqt. \quad (14)$$

If we substitute $\frac{dy}{dt}$, $\frac{d^2y}{dt^2}$, and $\frac{d^2y}{dx^2}$ from (14) in (12) and place, in the resulting equation, the coefficients of $\operatorname{Sin} mqt$ and $\operatorname{Cos} mqt$ respectively, equal to zero, we have two equations from which we find

$$D_{mn}=\frac{(c^2n^2p^2-m^2q^2)A_nB_m}{(c^2n^2p^2-m^2q^2)^2+m^2q^2\beta^2}, \quad (15)$$

$$E_{mn}=-\frac{mq\beta A_nB_m}{(c^2n^2p^2-m^2q^2)^2+m^2q^2\beta^2}. \quad (16)$$

[1] The motion of any system having one degree of freedom, maintained by a simply periodic force, is of the form $A \operatorname{Sin} qt+B \operatorname{Cos} qt$, in which $q=\frac{2\pi}{\tau}$, τ being the period of the force. The wire may be considered as a system having one degree of freedom so far as any one of the terms of (11) is concerned. Indeed, any system may be considered as having one degree of freedom so far as concerns the action of an impressed distributed force depending upon a single variable, provided that we ignore such motion as is *proper* to the system, such, that is, as is independent of impressed force.

Similarly, the maintained vibrations depending upon (13) are of the form

$$y = D'_{mn} \operatorname{Sin} npx \operatorname{Sin} mqt + E'_{mn} \operatorname{Sin} npx \operatorname{Cos} mqt, \tag{17}$$

in which

$$D'_{mn} = \frac{mq\beta A_n C_m}{(c^2n^2p^2 - m^2q^2)^2 + m^2q^2\beta^2}, \tag{18}$$

and

$$E'_{mn} = \frac{(c^2n^2p^2 - m^2q^2)A_n C_m}{(c^2n^2p^2 - m^2q^2)^2 + m^2q^2\beta^2} \tag{19}$$

Summing (14) and (17) for all values of m and n, we have for the total motion maintained by (11)

$$\begin{aligned} y = &\sum_m \sum_n [D_{mn} \operatorname{Sin} npx + D'_{mn} \operatorname{Sin} npx] \operatorname{Sin} mqt \\ &+ \sum_m \sum_n [E_{mn} \operatorname{Sin} npx + E'_{mn} \operatorname{Sin} npx] \operatorname{Cos} mqt. \end{aligned} \tag{20}$$

Let the motion of the wire be observed at the point x, so that for this point

$$y = \Sigma[G_m \operatorname{Sin} mq(t+\alpha) + H_m \operatorname{Cos} mq(t+\alpha)], \tag{21}$$

in which the G's and H's are known, and α is an unknown time interval. By expansion of $\operatorname{Sin} mq(t+\alpha)$ and $\operatorname{Cos} mq(t+\alpha)$, (21) becomes

$$\begin{aligned} y = &\sum_m [G_m \operatorname{Cos} mq\alpha - H_m \operatorname{Sin} mq\alpha] \operatorname{Sin} mqt \\ &+ \sum_m [G_m \operatorname{Sin} mq\alpha + H_m \operatorname{Cos} mq\alpha] \operatorname{Cos} mqt. \end{aligned} \tag{22}$$

This must be identical with (20), whence by equating coefficients, we have equations of the form

$$\sum_n [D_{mn} \operatorname{Sin} npx + D'_{mn} \operatorname{Sin} npx] = G_m \operatorname{Cos} mq\alpha - H_m \operatorname{Sin} mq\alpha, \tag{23}$$

$$\sum_m [E_{mn} \operatorname{Sin} npx + E'_{mn} \operatorname{Sin} npx] = G_m \operatorname{Sin} mq\alpha + H_m \operatorname{Cos} mq\alpha. \tag{24}$$

The number of such equations is equal to twice the largest value of m. The quantities c^2, l, and β being known, equations (23) and (24) involve the B's and C's and α as the only unknowns. We may, therefore, eliminate α, and calculate all the B's and C's in terms of one of them.

If the motion of the wire be observed before it becomes steady, then there will be, in addition to the above motion, a superposed proper motion of the wire, such that at the instant the alternating circuit is closed, the velocity and displacement of each point of the wire pertaining to this proper motion will be equal and opposite to the velocity and displacement pertaining to the maintained motion (considered as being already under way). In this case also the problem is easily solved.

PROBLEM III. — *A body of unit mass is maintained in vibration by a simply periodic force,* $F \operatorname{Sin} qt$*; required its amplitude and phase.*

Let $-\gamma x$ be the force, due to elasticity, tending to bring the body from a displacement x to its position of equilibrium, and let $-\beta\frac{dx}{dt}$ be the damping force which acts upon the body. Consider a fictitious force $-\frac{d^2x}{dt^2}$ equal and opposite to that required to accelerate the body in its motion. Then the four forces,

$$F \operatorname{Sin} qt, \quad -\gamma x, \quad -\beta\frac{dx}{dt}, \quad \text{and} \quad -\frac{d^2x}{dt^2},$$

are in equilibrium, and

$$\frac{d^2x}{dt^2}+\beta\frac{dx}{dt}+\gamma x-F \operatorname{Sin} qt=0. \tag{25}$$

For the graphical solution we assume that vibrations maintained by a simply periodic force are also simply periodic and of the same period as the force. Any consistent result to which this assumption leads will be a legitimate solution of the problem. Let X (unknown) be the maximum value of x; x being simply periodic; so also are γx, $\beta\frac{dx}{dt}$, and $\frac{d^2x}{dt^2}$ with maximum values γX, $q\beta X$, and q^2X respectively. The instantaneous values of x, $-\gamma x$, $-\beta\frac{dx}{dt}$, $-\frac{d^2x}{dt^2}$, and $F \operatorname{Sin} qt$ will be represented by the respective projections, on any stationary line, of a rigid system of lines representing X, γX, $q\beta X$, q^2X, and F to scale, and rotating at angular velocity q in the diagram, Fig. 1. These lines, γX, $q\beta X$,

q^2X, and F are respectively π, $\frac{3\pi}{2}$, 0, and θ (unknown) ahead of the line X. In order that the sum of the projections of γX, $q\beta X$, q^2X, and F may always be zero, the vector sum of these lines must be zero, so that $-F$ is the diagonal of the rectangle shown.

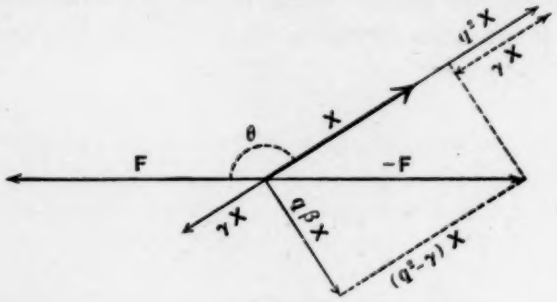


Fig. 1.

From this diagram we have

$$F^2=q^2\beta^2X^2+(q^2-\gamma)^2X^2,$$

or

$$X=\frac{F}{\sqrt{(q^2-\gamma)^2+q^2\beta^2}}, \tag{26}$$

and

$$\tan\theta=\frac{q\beta}{q^2-\gamma}. \tag{27}$$

These two equations determine the amplitude X of the vibrations, and the phase difference θ between the force and the vibrations. It is well to notice, what the diagram shows with great clearness, that any simply periodic force which maintains a body in vibration is broken up into two parts, one of which balances damping, and the other of which combines with elastic forces and determines the period of the vibrations.

In conclusion, we would call the reader's attention to the building-up process employed in problems I. and II., to the note to problem II., and to von Helmholtz's words concerning this third problem: "Hierdurch ist das Problem der erzwungenen Schwingungen gänzlich beantwortet." Perhaps the only case of maintained vibration which is not built up from this problem III.

is that of a string, forced by a tuning fork, which produces a periodic variation in the tension of the string — Melde's experiment. For a discussion of this problem see Rayleigh, *Philosophical Magazine* (5), Vol. 15, p. 229. Vibrations forced in a string by lateral *motion* of one of its ends also does not clearly fall in line with the above. For very good discussion of such, see A. Elsas, *Wiedemann's Annalen*, 23, pp. 161–173.

MINOR CONTRIBUTIONS.

The Thompson Physical Laboratory at Williams College.

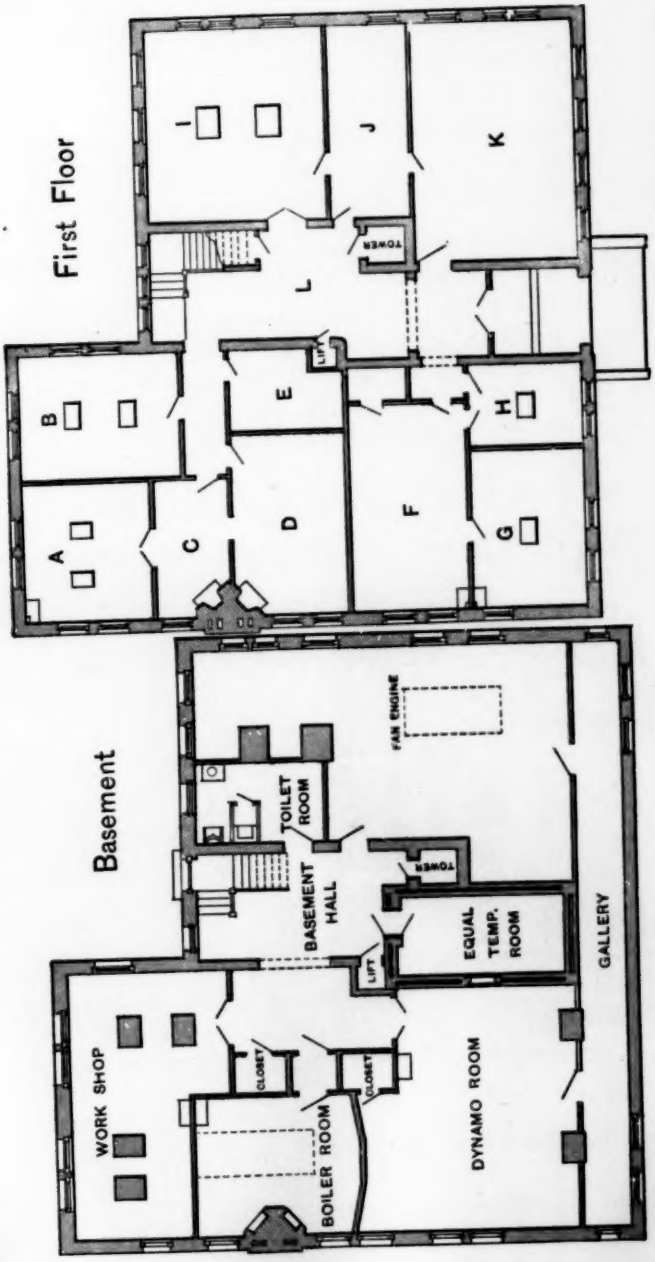
By Henry Lefavour.

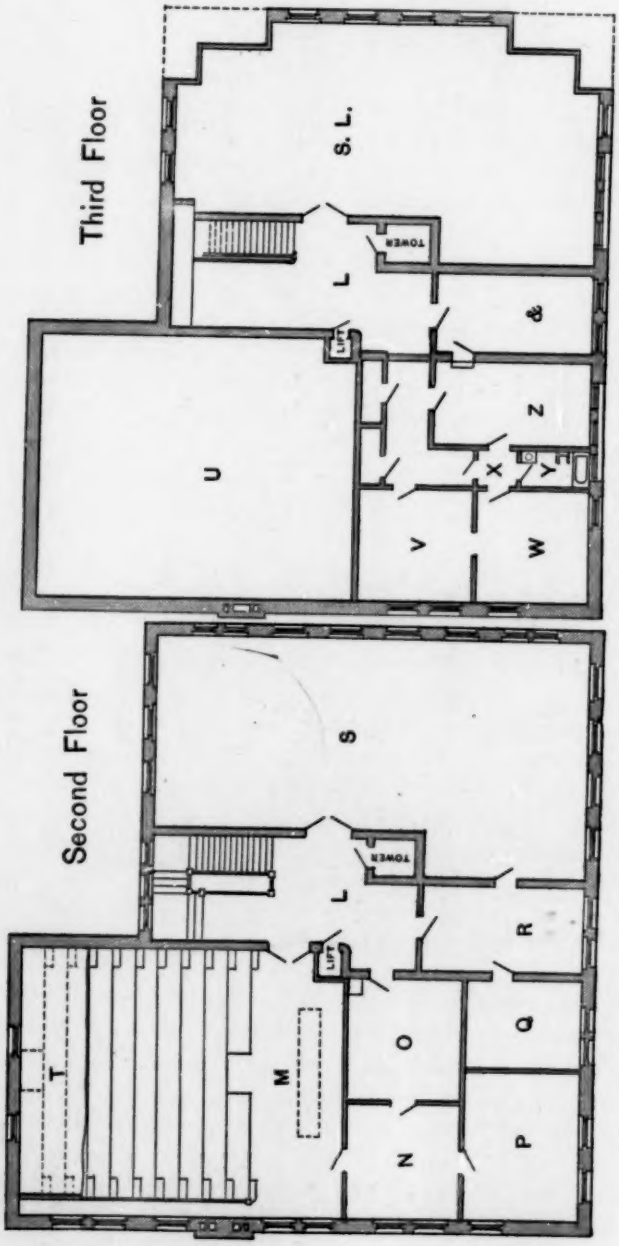
THE Thompson Physical Laboratory, in addition to two other buildings for the chemical and biological departments respectively, was built and presented to Williams College by Frederick F. Thompson, Esq., of New York City. The architect was Mr. Francis R. Allen, of Boston, and the principal contractors were Messrs. Haskell Dodge & Co., also of Boston.

The buildings are constructed of "rain-struck" brick, trimmed with

Fig. 1.

Warsaw bluestone, their front elevations of the same height, and planned with reference to a symmetrical arrangement. The physical laboratory (Fig. 1) is the easternmost of the three, and is separated from the second building, the chemical laboratory, by an interval of about 50 feet. It has a frontage of 80 feet on the north, a depth of 60 feet on the west, while


First Floor
A
B
C
D
E
F
G
H
I
J
K
L
TOWER
LIFT
Basement
WORK SHOP
CLOSET
BOILER ROOM
CLOSET
BASEMENT HALL
TOILET ROOM
FAN ENGINE
TOWER
LIFT
EQUAL TEMP. ROOM
DYNAMO ROOM
GALLERY


Third Floor
S. L.
TOWER
L
LIFT
&
Z
U
X
Y
V
W
Second Floor
S
TOWER
L
LIFT
R
T
M
O
Q
N
P

the eastern half is carried back by a projection to a total depth of 77 feet. The heights of the different floors vary, that of the basement being 8 feet in the clear; the first and third floors, 10 feet and 6 inches; and the second floor, 12 feet and 10 inches. The lecture room has the height of two floors for the most part, but all the rooms of the third floor lose in height near the walls because of the shape of the roof.

The interior is finished in hard pine, the walls and ceilings are plastered with King's Windsor cement, and are tinted with a light cream color. The building is divided into three sections by brick partitions, which enclose the hallways, and the floor timbers rest on iron girders, which in turn are supported by these walls. Eight brick piers with stone caps rise from the foundations into various rooms of the first story, and all workrooms are amply provided with slate shelves imbedded in the brick walls.

The laboratory is heated by direct radiation and by forced draught; the former method being employed in the small rooms, the latter in the larger and more crowded rooms, thus securing thorough ventilation at the same time. The steam is brought from the chemical laboratory at high pressure, and is distributed to the engine which operates the Sturtevant blower, and to a reducing-valve, from which it is carried at the ordinary low pressure to the radiating coils. Each workroom is supplied with hoods, water, gas for illuminating and heating purposes, electric current, and with cases for apparatus.

The instruction in physics in the college contemplates a required course in general physics for the sophomores, to be carried on by lectures, recitations, and elementary laboratory work, a year of more advanced general physical measurements (which is elective to the juniors), and a third year's course in special problems or thesis work for those of the senior class who may have developed a special interest in the subject. The arrangement of the rooms, therefore, includes a lecture room and the accessory apparatus rooms, an elementary laboratory, a general laboratory, and a few smaller rooms to be equipped for special investigations and measurements, as well as for the use of professors and assistants. The accompanying floor plans show the situation of the different rooms.

The basement contains a workshop, for which power is to be supplied by water and electric motors, a boiler room containing a 20 h.-p. boiler, used exclusively to furnish power for the engine, a toilet room, a room for the Sturtevant blower and the heating coils, the room to be used also for unpacking and storing cases, a room with double walls, in which the temperature can be maintained fairly constant, a long gallery extending the length of the building, for experiments requiring a long uninterrupted space, closets for chemicals and the storage battery, and a dynamo room. The dynamo room is equipped with a 15 h. p. Westinghouse engine, a

5 k. w. incandescent dynamo, two experimental motors, and the usual regulating and measuring apparatus.

On the first floor are the professor's office, *C*, with his private laboratory, *A*, on the one side, and the library of the laboratory, *D*, on the other. By permission of the Trustees, the books and periodicals that are of especial interest to the several scientific departments are now kept in the respective laboratories, and it has already proved to be of the greatest advantage to have the books so accessible. *B*, *H*, *F*, and *I* are the rooms to be devoted to advanced physical work, *B* to optical or spectrometric measurements, *F* to photography and chemical physics, and *I* to electrical testing. In the last room and in its vicinity an effort has been made to avoid the use of iron in any way that would make the magnetic field unusually variable. The small room *H* is reserved for any general research work. A recitation room, *K*, with seats for fifty, is at the right of the entrance, and next to it is the coat room, *J*. *E* is a storeroom for laboratory supplies, and *G* is the assistant's private laboratory.

On the second floor, *S* is the general laboratory for the juniors. It is furnished with movable tables with slate tops, with wall shelves, a bookcase, a case for chemicals, and a set of lockers for the students. Leading from this is the laboratory apparatus room, *R*, which also serves as the assistant's office. *Q* is used for acoustical measurements or for any work that requires isolation. Next to this is the photometric room, *O*, without windows and with blackened walls. The lecture room, *M*, will seat about 140. It is furnished with the Heywood chair, with tablet, arm, and hat rack. The tiers rise with a general slope of one foot in three. The lecture table has the usual conveniences of water, gas, and electricity, and in addition there are pipes for bringing compressed air, oxygen, and hydrogen from the basement. There are places for the projecting lantern in the rear of the room and in front of the lecture table, and for both positions there are permanent plaster screens on the wall behind the table. The room is lighted by eight windows, and these may be darkened by black, opaque shades, reinforced by shutters when there is need of perfect darkness. These shades can be operated from three points, but it is possible to have them all connected with a water cylinder. For the present, whenever there is need of interrupting the projections, the room will be lighted by electricity. The sun's rays may be brought into the room through either the east or the south windows. The room is connected with the dynamo room by a speaking-tube, and the whole building is well supplied with electric bells. Adjoining the lecture room are the apparatus rooms, *N* and *P*, and the space under the seats, *T*, is also used for storing apparatus.

On the third floor is the elementary laboratory, *S.L.*, of nearly the same size as the general laboratory below it, but it is more simply equipped.

The remainder of the floor is taken up with two suites of assistants' rooms, *V–W* and *Z–&*, the bath room, *Y*, and the upper part of the lecture room, *U*. Provision in case of fire is made by having the stairways of iron with stone treads, and by placing two fire escapes on the outside. A brick-walled shaft, marked "Tower" on the plans, extends from the basement to the roof, and affords facilities for pendulum and other experiments requiring height or depth, and a small lift serves to transport apparatus from the basement to the upper floors.

As is frequently the case, there have been restrictions which prevented the fullest realization of the original plans, and larger experience always suggests improvements, but thus far the building has shown itself in its six months of use to be admirably adapted to the demands made upon it.

REACTION VELOCITY AND BOILING-POINT.

BY J. E. TREVOR AND F. L. KORTRIGHT.

IT is well known that the action of acids upon aqueous solutions of cane sugar consists in a transformation of the cane sugar as such into a mixture of dextrose and levulose, the so-called invert sugar, a transformation represented by the chemical equation

$$C_{12}H_{22}O_{11} + H_2O = C_6H_{12}O_6 + C_6H_{12}O_6.$$

The acid present remains entirely unaffected. During this process the molecular concentration of the solution, *i.e.* the number of gram-molecules of dissolved substance per liter, suffers a continuous change, increasing to twice its initial value as the chemical transformation approaches completion.

In consequence of this the change, with the time, of any property of the solution which is dependent upon the molecular concentration, furnishes a ready means for direct observation of the velocity with which the chemical reaction in question takes place. The decrease of the vapor pressure of the solution, the depression of its freezing-point below that of water, the rise of its boiling-point, and the solubility of its solvent in some second liquid are such properties, each being directly proportional to the molecular concentration, or very nearly so. It follows, then, for example, that the boiling temperature of a sugar solution must rise as the inversion proceeds, and that the velocity with which the boiling-point is displaced measures the velocity of the reaction. This furnishes a very convenient method for observing the latter.

From the classical researches of Wilhelmy [1] and of Ostwald,[2] it is known that the velocity with which cane sugar is inverted by an aqueous acid is, at any moment, proportional to the concentration of the sugar remaining unchanged, and to the acidity of the acid; *i.e.* to the concentration of its electrolytically dissociated, free hydrogen ions. For an acid of a constant concentration this latter quantity remains constant. The relations obtaining are expressed by the differential equation

$$\frac{dx}{dt} = k \cdot a(s - x),$$

in which x represents the amount of sugar which is inverted at the time t, s the initial concentration of the sugar, and a that of the acid; k is the proportionality factor. For s is to be substituted the rise of boiling-point of the original solution, and for x the temperature increase at the time t over that due to the sugar and acid employed. Integrating from o to t, and noting that for $t = 0$, $s - x = s$, there results

$$\frac{1}{t} \cdot \log \frac{s}{s - x} = k \cdot a.$$

For the experimental determination of this constant ka, a small quantity (0.220 grams) of succinic acid was added to a boiling solution of cane sugar contained in a Beckmann boiling-apparatus,[3] and successive readings of the boiling temperature of the liquid were taken. The sugar solution contained 33 grams of granulated sugar in 70 grams of a very pure distilled water, and was, therefore, approximately a normal solution. The thermometer employed was graduated to hundredths of a degree, and was read with a lens. The results obtained are as tabulated:

[1] Poggendorf's Annalen, 81, p. 413 (1850).
[2] Journal für praktische Chemie, 29, p. 385 (1884).
[3] Zeitschrift für physikalische Chemie, 4, 532 (1889).

TABLE.

$s = 0^{\circ}.759$. $a = 0^{\circ}.008$.

t	x	$\frac{1}{t}\log_{10}\frac{s}{s-x}$
6 minutes	0°.114	0.0118
9	0°.171	122
16	0°.275	122
19	0°.311	121
22	0°.342	118
25	0°.383	122
28	0°.412	121
31	0°.442	122
34	0°.463	120
37	0°.485	120
40	0°.506	119
43	0°.523	118
85	0°.683	118
Mean		120

The observed constancy of the characteristic logarithmic expression is as satisfactory as might have been anticipated, especially since no correction was made for the influence of fluctuations in the barometric pressure. During the experiment the barometer fortunately remained very nearly constant at 754.5 mm. At the last reading, after the lapse of an hour and a half, the table shows $\frac{683}{759}$, or 90 per cent, of the total sugar present to have been inverted.

The accuracy of the method might be materially increased by carefully connecting the boiling apparatus with a large, empty bottle, and closing all connection with the outside atmosphere at the beginning of each experiment. In this way the work would be conducted under an unvarying pressure, and from simultaneous barometric readings the observed boiling-points could be reduced to a normal condition of pressure. By making the experiments with, perhaps, three different concentrations of the sugar solution, and extrapolating for the velocity constant at zero concentration, the effect of the sugar in altering the electrolytic dissociation of the acid, and consequently the velocity of inversion, would be very largely eliminated. The ratio of each constant so obtained to that found with a very dilute, *i.e.* completely dissociated, solution of hydrochloric acid would then give in each case the relative concentration of the free hydrogen ions of the acid employed.[1]

[1] See Trevor, Zeitschrift für physikalische Chemie, 10, 321 (1892).

In this way it would become possible to detect, with a very considerable accuracy, the degree of electrolytic dissociation of the acids at high temperatures, and from the data so secured, by a comparison with the very extended and accurate determinations made by Ostwald[1] at 25°, the change of this dissociation with changing temperature would become very much better known than is at present the case. This knowledge would render accessible, by direct calculation, that important and but slightly known quantity, the heat of electrolytic dissociation, which determines the influence of temperature change upon the dissociation of electrolytes.

CORNELL UNIVERSITY, January, 1894.

DYNAMICS OF THE ETHER.

BY A. E. KENNELLY AND REGINALD A. FESSENDEN.

THE fundamental equation in the dynamical theory of the propagation of disturbances in an infinite isotropic medium such as the ether may be, is

$$\rho\ddot{v} = F\rho + \nabla P,$$

where v is the vector velocity, ρ the density, F any external force subject to a potential, and P the impressed force per unit area.

The existing dynamical theory of ether, therefore, assumes that ether possesses inertia like material media, and the same equation is also fundamental in determining the propagation of disturbances in material media such as liquids or gases.

But it is possible that the inertia, which is the attribute of matter, is really an effect due to the ether pervading matter, and not in the matter itself, or in other words, it is possible that if a body could be completely divested of its internal ether, and be then placed in a space devoid of ether, it would possess no inertia.

If this should be the case, the property of inertia could not be assumed to be possessed by the ether, itself considered as matter, without further justification, and the present fundamental dynamical equation could no longer be claimed.

The conclusions which follow from this equation, when applied to the ether, are known to involve the existence of two independent systems of waves from any source of disturbance: the first, a wave of distortion and rotation without compression, moving with velocity $\sqrt{(\nu/\rho)}$; and the second, a wave of simple compression like a sound wave in air, proceeding with a velocity $\sqrt{(\sigma + \frac{4}{3}\nu)/\rho}$, where σ is the resistance to com-

[1] Zeitschrift für physikalische Chemie, 3, 170 (1889).

pression, and ν the rigidity. The former accords with the behavior of light, while there is experimental evidence that the latter wave is either totally absent or has only a very diminutive existence. In order to account for the absence of this sound-like wave, it has been suggested that its velocity of propagation is either enormously great, or exceedingly small, which requires that the resistance σ of the ether to compression should be either very large, compared to its resistance to shearing ν, or should have a certain negative value; *i.e.* that the medium would have to be prevented from exploding.

It appears, therefore, that both these difficult suppositions can be avoided if the inertia possessed by matter is due to reaction from distortion of the ether within that matter, for in that case the equations of motion hitherto assumed for ether would need modification.

That such distortion takes place in the ether within a moving body, seems indicated by the experiments of Fizeau and Michelson; that it is limited, at least sensibly, to the interior of the body, appears to be shown from the recent experiments of Lodge, while that reaction follows the distortion of ether is evidenced by the phenomena of light.

THE LEYDEN JAR AS A STORAGE BATTERY.

BY S. T. MORELAND.

FOR some years I have been in the habit of showing the following experiment to my classes. Take two large Leyden jars and connect their inner coatings respectively to the two poles of a ten-inch Toepler-Holtz electrical machine, their outer coatings being connected to each other. Separate the poles of the machine so as to get as long a spark as possible, and charge the jars nearly to the point of discharging. Now remove the belt and give the plate a slight backwards motion, and it will continue to revolve backwards for several seconds, and the jars will gradually lose their charge, the energy being used of course to drive the plate. For success in the experiment, it is necessary to have the bearings of the plate nicely cleaned and oiled, and to see that the brushes do not cause too much friction by rubbing against the buttons on the plate.

WASHINGTON AND LEE UNIVERSITY,
Oct. 5, 1893.

The Times of Descent along Chords of a Circle.

By James S. Stevens.

THIS note deals with a simple experiment to show that the times of descent down the chords of a circle are equal to each other and to the time of falling through the vertical diameter. I have devised a simple method of demonstrating the above proposition by the use of two pieces of lath joined together by a hinge. As it gives uniformly good results, I submit it to physical laboratory workers.

AB and *AC* (Fig. 1) are two pieces of wood of equal length hinged at *A*, so that *C* can trace the curve *CD*.

BAC can be made any angle up to 90°; and if a string equal to the radius *AO* be fastened at *O* and the other extremity be joined to the moving bar when at *AC''* (say), then *AA''* will be a chord of the circle. Now if the upper surface of *AC''* is grooved so that a marble can roll down, and if a tin box be fastened at *A''*, marbles dropped through *AB* and rolled through *AA''* will reach their destination simultaneously.

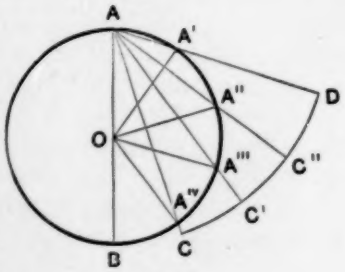


Fig. 1.

This will, of course, be true when the point of contact between the string and bar is at *A'*, *A'''*, or *A''''*.

Maine State College, Dec. 2, 1893.

NEW BOOKS.

The Todhunter-Pearson History of Elasticity.[1]

Dr. Isaac Todhunter, at his death in 1884, at the age of sixty-four, left, in an unfinished manuscript, the fourth of his celebrated Mathematical Histories, — "The History of the Theory of Elasticity." This manuscript consists of two parts: the first is a history of the theory of elasticity; and the second, a mathematical treatise on the theory of the "perfect" elastic solid, is supplementary thereto. This manuscript contains analyses of most of the important mathematical memoirs up to 1870; it had been once rewritten, and then again revised by Dr. Todhunter. It was sent to Professor Cayley[2] for examination, and was pronounced of the same grade of excellence as the previous histories, and fairly complete. The task of completing and editing was soon afterwards entrusted, at the suggestion of Dr. Routh, to Professor Pearson by the Syndics of the University Press.

Volume I.,[3] pp. 924, Gallilei to Saint-Venant, 1639 to 1850, appeared in 1886. In the preparation of this volume Professor Pearson, "from his own account, devoted sufficient labor to have enabled him to complete it *ab initio*," without much exceeding the strict duties of editor. An extract from Vol. II. appeared in 1889 under the title "The Elastical Researches of Barré de Saint-Venant,"[4] pp. 286, of which Professor Greenhill says, it is "a monument of painstaking energy and enthusiasm!" The History now reaches completion by the appearance (1893) of Vol. II., Parts I. and II., pp. 762 and 558. This carries the analysis of individual memoirs completely to the year 1860, and the work of Saint-Venant and Boussinesq, Rankine and Lord Kelvin, F. Neumann, Kirchhoff, and Clebsch, up to the present date. The plan of the History has been considerably modified by Professor Pearson. It was decided to print only the historical part of Dr. Todhunter's manuscript, so that the gaps thus left had to be filled; Professor Pearson undertook — wisely, it seems — to widen the scope of the

[1] *A History of the Theory of Elasticity and of the Strength of Materials* from Gallilei to the present time. By the late Isaac Todhunter, D.Sc., F.R.S. Edited and completed for the Syndics of the University Press by Karl Pearson, M.A., Professor of Applied Mathematics, University College, London. Cambridge, 1886 and 1893.

[2] See obituary notice of Dr. Todhunter. Proc. R. Soc., Vol. 37, p. xxvii.

[3] Review by Professor A. G. Greenhill, Nature, Feb. 3, 1887.

[4] Review by Professor A. G. Greenhill, Nature, March 20, 1890.

History so as to include physical and technical elasticity; and the adoption of uniform terminology and notation became an evident necessity during the progress of Vol. I., which was printed in instalments. This uniformity is found in all but the first three chapters and part of Chapter IV., in which, indeed, terminology and notation are matters of some indifference. The changes thus introduced into Vol. I. are slight; in Vol. II., however, but little of Dr. Todhunter's manuscript remains, although the substance of most of it is no doubt incorporated. The History, judging mainly from the chapter (XIV.) devoted to Lord Kelvin and P. G. Tait, is thoroughly and well done, and constitutes indeed a "*Repertorium* of elasticity rather than an *Historique Abrégé*." It is pleasing to note the extent to which Professor Pearson has been encouraged and assisted in the execution of the work; in the way of careful verification of the analysis throughout (not mere proof-reading); the correction and revision of certain chapters by those in position to make their revision more or less authoritative; and by friendly aid and suggestion from several hands. This is an important feature of the work, and must go far towards giving permanent value to the enormous amount of labor which Dr. Todhunter and Professor Pearson have devoted to the History.

The general value of such compendious writings is no doubt in proportion to the labor they involve, but to us Americans in particular their value cannot be overestimated, since they enable us to outlive, in a measure, the inconvenience of new and incomplete libraries. Yet many of us who study to teach will be slow to feel the benefit. We should keep in mind the "true ends of knowledge and endeavor, not after it for curiosity, contention, or the sake of despising others, nor yet for profit, reputation, power, or any such minor consideration, but solely for the occasions and uses of life." Study is the cosmic thread of action, otherwise dark, frantic confusion and chaos without end.

W. S. FRANKLIN.

On Vision.[1] By WILLIAM FERREL.

No one ever associates the name of the late Professor Ferrel with physiological optics. He was one of those who never attacked a subject except by methods exclusively his own; and never was it left without the attainment of truth new to himself. His modesty was such as might almost be deemed blamable; and the present essay would have remained forgotten but for the faithfulness of a scientific friend who will incorporate it in a forthcoming volume of collected memoirs. It was published in 1855 in the Nashville *Journal of Medicine and Surgery*, and quite naturally

[1] To be republished in a forthcoming collection of memoirs.

attracted but scant attention at that time, for the subject was one of little interest to practitioners of the art of healing. It was one of the first papers ever published by its author, whose subsequent work established his position as a mathematician and meteorologist of the highest rank.

To appreciate this research it is necessary to know something of Ferrel's early life. Amid humble surroundings his opportunities for systematic education had been slender, and there was no access to scientific libraries or apparatus. Throughout a period of twenty years, beginning with 1838, he earned a livelihood by teaching school, for the most part in small western towns. His residence in Nashville was included between 1854 and 1858. This fact accounts for his recital of a few discoveries which were none the less his own, even if he was anticipated by others whose writings had not been accessible to him.

Ferrel's treatment of the subject of vision is plainly that of one accustomed to mathematical rather than metaphysical methods. He indulges in no speculation, and apparently consults no authorities until after he has reasoned out his problem analytically and tested his conclusions by experiment. On his first page he refers to the possibility of having been anticipated to some extent by Wheatstone, whose papers he had been unable to see. Wheatstone's communication on the "Physiology of Vision" had been given to the Royal Society in 1838, but no general impulse to the study of this subject was received until 1849, when Brewster's refracting stereoscope was brought before the public, while the new art of photography was rapidly developing so as to make it possible to construct stereographs more accurately and easily than by hand. Ferrel was evidently unaware of the warm controversy between these two eminent physicists on the question of priority regarding the invention of the stereoscope, and his investigation was undertaken independently of what they had done.

The discussion is begun with a consideration of the law of visible direction for a single eye. The theory of Brewster, then generally accepted, was that we see every object in the direction of a perpendicular to the retinal point on which the object is depicted. Ferrel shows mathematically that this hypothesis is applicable only to a special case in which the visual line passes through both the optical center of the refracting combination and the geometric center of the eyeball, supposing the latter to be spherical. If the surface of the cornea be assumed spherical, its center of curvature does not coincide with either of the two centers just named. These facts being taken into account, it is shown by the laws of refraction that no object seen by indirect vision can appear in accordance with Brewster's hypothesis, which requires that the visual line shall pass through the center of the eyeball. On the contrary, an object must appear in the direction of the optical center of the refracting combination,

which may be assumed without sensible error to be that of the crystalline lens. This direction is in general oblique to the retinal surface impressed. The following experiment, devised by Ferrel, serves as an excellent and decisive test, to show that by indirect vision the apparent direction of an object is nearer to the axis of vision than is its real direction. Let the eye be placed just above the corner of a rectangular table, and directed along the bisector of the right angle. Then by indirect vision the two adjacent sides next the eye appear to form with each other an angle considerably less than a right angle. The illusion is quite striking when perceived for the first time.

An inquiry is then made regarding the locus of distinct vision with a single eye; for example, in a horizontal plane. This is shown to be a circle passing through the optical center of the crystalline, but with variable radius, depending upon the distance for which the accommodation of the eye is adapted. To test this experimentally is not so easy as in the previous case, for not only does the accommodation involuntarily vary, but it is impossible to assign a definite limit at which distinct vision begins to become indistinct, especially when the vision is indirect.

The author next treats of the principles of binocular vision. The circle of distinct vision for a single eye must pass through both optic centers, since by experiment it is found that the accommodation of the two eyes is always the same at the same moment. As to the mechanism of accommodation no conjecture is offered. The now classic experiments of Donders and Helmholtz were at that time in progress, but nothing was known about them in America. The common axis of vision for the two eyes the author considers to be the bisector of the optic angle between the two visual lines which meet at the point of fixation. No mathematical reason for this is or can be assigned, but assuming as a fact that the apparent standpoint of the binocular observer is midway between the two eyes, and accepting the theory of corresponding retinal points, it is easily shown geometrically that all objects in a given field, when viewed with both eyes, must appear as if viewed with one, so far as position is concerned, and that any object within or beyond the circular locus of fixation must appear double. This subject is developed somewhat in detail, and a number of experiments are described, some of which demand more muscular training of the eyes than is possessed by the majority of readers, though it is not hard to acquire. The judgment of distance, whether with a single eye or both eyes, is in reality somewhat complex; and the apparent position of an object is often determined largely by expectant attention and imagination, when vision becomes in any way abnormal, so that two good observers may honestly disagree. The author expresses the conclusion that the mind, even in monocular vision, judges of distance by visual

triangulation, the interocular distance serving as a base line. The uncertainty of judgment is, of course, less in binocular than monocular vision. A formula is deduced for the determination of the smallest space upon the retina sensible to the mind. From experimental data this diameter is computed to be 0.000048 of an inch, which corresponds to about 16″ of arc, taking the distance to optical center of crystalline as radius. This value is much smaller than that commonly accepted, 1′ of arc. Ferrel shows by application of his formula that at the distance of about a mile the apparent difference of distance between the sun or moon and an intervening cloud must vanish. If the ordinary value, 1′ of arc, be taken, the distance is less than a mile. In either case all judgment of distance vanishes when the observer is restricted to visual triangulation.

The principles set forth are now applied to vision with the stereoscope, and the apparent position of the stereoscopic image is determined by intersection of visual lines. It is shown that with the unaided eyes either orthoscopic or pseudoscopic effects may be obtained by varying properly the relation between the visual lines. In relation to the diminished image and pseudoscopic effect attained by cross-vision Ferrel remarks, "this, so far as I know, has never been noticed heretofore." The discovery was thus wholly original with him, though he had been anticipated by both Wheatstone and Brewster. The latter especially describes various phenomena of cross-vision, and exhibits much geometric ingenuity in the work of explanation, though he falls into error because dominated by geometric ideas which in reality cannot be fully applied to binocular phenomena. The present writer showed some years ago (1881) that these binocular effects are attainable with the visual lines either parallel or divergent, a condition that forbids all intersection of visual lines for the determination of the apparent position of stereoscopic images. When the visual lines are convergent there is generally a rough accordance between the appearance and the result attained by applying a mathematical formula. Indeed, it may be said that mathematics fails when applied to visual judgments, though fully applicable to the eye as an optical instrument.

Ferrel evolves analytically a method of making stereographs by use of rectangular co-ordinates, resulting in a general formula appropriate for either orthoscopic or pseudoscopic effects. Brewster had already discussed this subject geometrically, but his discussion lacked the simplicity and generality of Ferrel's.

The essay is closed with a consideration of the views on the physiology of vision held by such authorities as Carpenter and Berkeley. The author, expressing himself with due caution, inclines to the belief that the existence of corresponding retinal points cannot be owing to habit, but to some natural constitution of the eyes. He notices somewhat fully Berkeley's

theory of vision, that all visual sensations are refinements of the sense of touch, and shows that it is untenable; that lines and angles have something to do with our visual perceptions, and that the place occupied by mathematics cannot be removed by psychological considerations from the philosophy of vision.

The entire investigation is worthy of the careful, exact, and thorough man of science, whose achievements in other fields made him an undisputed authority, respected and honored by all whose attainments entitled them to an opinion of his work.

W. Le Conte Stevens.

Inventions, Researches, and Writings of Nikola Tesla. By Thomas Commerford Martin. 8vo, 500 pp. The Electrical Engineer, New York, 1894.

This book for the first time affords the reader an opportunity to make a continuous and consecutive study of Mr. Tesla's work. Mr. Tesla has been more of a student and inventor than a writer and lecturer. He finds the laboratory more congenial than the lecture-room, and takes a much deeper interest in the prosecution of new lines of research and the development of new facts, than in the presentation to the public of the results of his work. As a consequence, Mr. Tesla's lectures and published papers have appeared at irregular and rather long intervals, and are scattered through the technical journals and in separate pamphlets.

Those interested in Mr. Tesla's work are greatly indebted to Mr. Martin for collecting as he has done into one volume all of Mr. Tesla's published papers, and especially for the connecting links and summaries that, as Mr. Tesla's intimate personal friend, he was so well qualified to supply.

The work begins with a brief biographical sketch, and we find that, as usual with great inventors, Mr. Tesla's fertile brain was even, while he was a student, developing new ideas, and it is interesting to note that the germ of the idea of the rotating field was conceived before the beginning of 1880.

Part I. deals with the rotating field apparatus, beginning with Mr. Tesla's paper read before the American Institute of Electrical Engineers in May, 1888. This paper was a revelation, to most of those present at that meeting, of an entirely new departure in alternating current work. The rotating field principle has since become familiar to electrical engineers through the published accounts of experiments conducted in this country and abroad, and numerous workers have entered the field. Simple as the principle seemed when first employed by Mr. Tesla, it was found when the attempt was made to apply it to the construction of large motors, that, contrary to the usual experience with continuous current machines,

the large motors were less efficient than the small ones, especially when operated at high frequencies. With the talent brought to bear upon the subject, sources of loss have been discovered and eliminated, and the motors improved until now they stand nearly on a par in efficiency with continuous current motors.

The chapters which follow in Part I. describe a great variety of forms of rotary field motors. In some the difference of phase is obtained on branches from a single circuit by the proper proportioning of resistance and self-induction. In others, a "magnetic lag" in peculiarly formed polar-projections serves to produce the rotary effort by the use of a single alternating current. Many of these forms could hardly be developed into efficient commercial machines, but they are curious examples of the working of alternating currents, and are especially interesting here as showing the extreme fertility of Mr. Tesla's mind.

Part II. relates to Mr. Tesla's high frequency experiments. In it are published in full Mr. Tesla's three lectures delivered respectively, at New York, May 20, 1891; at London and Paris, 1892; and at Philadelphia and St. Louis in 1893. The lectures are preceded by a review by Mr. Martin.

It is to be regretted that this review is not more discriminating. Instead of pointing out what properly belongs to Mr. Tesla, and what had already been done by other experimenters, he leaves the reader to infer that everything described is Mr. Tesla's work. The claim for novelty of the experiments seems, in some cases, to be unduly exaggerated. The operation of electrical devices by means of one wire is spoken of as though it were something before unheard of; and yet, how does it differ in principle from the old method of measuring the capacity of a condenser by charging it through a galvanometer? One wire from the battery passes through the galvanometer and terminates in the condenser. Close the key, and a current rushes through the galvanometer coil into the condenser and swings the needle. Substitute an alternating potential for the constant potential of the battery, and an alternating current would flow through the galvanometer coil.

The writer more than ten years ago showed that the needle of a galvanometer would be deflected by the rush of electricity into the coil when a glass rod charged by rubbing with a silk handkerchief was brought to one terminal, the other terminal having no connection with anything whatever. Deflection in the opposite direction was caused by charging the coil through the other terminal. Deflection was also produced by discharging the coil by one of its terminals. The coil used in this experiment was very thoroughly insulated, and held the charge given to it for some time. This was a case of an electrical device actuated through one

connection only, the generator being a glass rod and a silk handkerchief. The writer did not consider it anything wonderful; in fact, it was just what he expected would take place in a coil sufficiently well insulated, and having a sufficient number of convolutions.

How are we to understand the statement on page 120 that Mr. Tesla has demonstrated, that for the production of light-waves, "primarily," electrostatic effects must be brought into play? It does not appear that Mr. Tesla has produced light-waves *primarily*, in any way.

But the writer does not wish to be understood as finding nothing except for criticism in Mr. Martin's summary; far otherwise. The reader will find it an excellent *résumé* of the results portrayed in Mr. Tesla's lectures, and will find it very useful, even after having read the lectures themselves.

Referring to the experiments exhibited by Mr. Tesla in his three lectures, their novelty seems to the writer to consist more in the brilliancy and magnitude of the results than in the development of anything radically new. By this he does not mean in any way to belittle the value of Mr. Tesla's work. His experiments are no less a revelation if they only exhibit old phenomena in a new light, and demonstrate experimentally facts that were before only theoretically known. By means of a high frequency alternating machine, and still further by the extremely rapid Hertzian oscillations, the old induction coil is made to exhibit some wonderfully brilliant phenomena. Geissler tubes glow with unwonted brilliancy. In Crookes' tubes, in place of heating to redness only a tiny piece of platinum, a whole lamp filament or a block of carbon is rendered incandescent. In consequence of the high frequency, great differences in the phenomena are produced by exceedingly small changes in capacity, and many of the experiments strikingly illustrate this fact.

In the title to Mr. Tesla's New York lecture we find that he proposes to deal with high frequency phenomena "and their application to methods of artificial illumination." It is in this last that the practical engineer is specially interested; and the first question that occurs to him is, what is the efficiency of the methods hinted at by Mr. Tesla? [We do not find the methods more than hinted at in these lectures, and there is nowhere any light thrown upon the question of efficiency.] Knowing as we do the losses that accompany the use of high frequencies and high potentials, such as those due to disturbances in surrounding media, and currents in conductors due to their capacity, it would be interesting to learn what method Mr. Tesla may have in view for eliminating these. Mr. Tesla speaks of using an electrostatic screen for a conducting wire, and remarks that such a screen would have the same effect as removing all objects to an infinite distance. The writer does not know of any such screen.

A metallic sheath would serve as a screen only if its own potential were

maintained constant, and that could only be if it were in perfect conducting communication with the earth. The effect of any such sheath, whether connected to earth or not, would increase the capacity of the conductor.

The scheme of lighting by means of isolated vacuum tubes in a rapidly alternating electrostatic field is an attractive one, but the plates forming the boundaries of such a field would have considerable capacity. To charge them to a potential of a hundred thousand volts would require a considerable quantity of electricity, and if this charge were reversed many thousand times per second, the transfer of that quantity of electricity would mean a considerable current, and this current would be continually flowing whether any work were being done or not.

To be efficient it would be necessary that the capacity of the system when the lights were not in use should be extremely small, and should be largely increased by bringing them into action. Is it likely that such a relation could be achieved?

In the single wire distribution about which so much is said in the lectures, the result is only reached by a very accurate adjustment of frequency, potential, and capacity. Supposing the question of efficiency settled, would not such a system still be inapplicable for an extended distribution where lamps are to be turned on and off, on account of the nicety of the adjustments required? Would not the balance be destroyed with every lamp turned off or on. Is it not true that in any system involving the use of very high potentials and high frequencies capacity becomes a most important factor, and that small changes in capacity may interfere seriously with the working of the system?

In closing his London lecture Mr. Tesla tells us that his screened cables will not be made, "for ere long intelligence — transmitted without wires — will throb through the earth like a pulse through a living organism." But how many people could use the earth at once? Would not the telephone girls' frequent answer, "line in use," have to be accepted in that case as at least plausible?

In the Franklin Institute lecture a number of pages are given to the subject of resonance. The writer is not sure that he understands fully the statements and illustrations in that part of the lecture, and will only refer to one or two points.

The statement that "pure" resonance effects are impossible in nature evidently means simply that motion as we have to deal with it cannot be maintained without the constant consumption of energy. This is a fact perfectly well known, and is, of course, true of vibratory motion as well as of any other.

A body acted on by a force ceases to increase in velocity when the

resistances it meets with equal the force applied. A vibrating body kept in motion by successive small impulses derived from another body which vibrates in unison with it, which is all there is of resonance, ceases to increase in vibration when the resistances it meets with equal the impulses, or, in other words, when the loss of energy during each vibration is equal to the energy received during the same time. If the energy of the receiver is employed for some useful purpose, it must, of course, receive from the original vibrating body as much energy as is used; otherwise it will cease to vibrate.

When Mr. Tesla says that in "electrical vibration it is of enormous importance to arrange the conditions so that the vibration is effected with the greatest freedom," it is the same as saying that the waste of energy must be reduced to a minimum, and this is just as true when electrical energy is transferred in other ways.

It is also just as important that the transfer should be efficiently effected.

The case of the resonant tuning-forks, where one is sounded and imparts its vibrations to all the surrounding air, a small portion of which air imparts its vibration to the other fork, is an example of a very inefficient way of transferring energy. The energy of the first fork is mainly frittered away in imparting useless vibration to the surrounding air. If the energy of the first fork could be efficiently transferred, the vibration of the second might be maintained in mercury, and if agitation of the mercury were the object to be attained, it might be efficiently effected; and yet the means of transferring the energy from one fork to the other might be such that no effective vibration could be imparted unless the two forks were tuned to unison.

In short, when energy is to be transmitted by means of periodic motions, it may be desirable or convenient to use receivers having a definite vibration period of their own, in which case they must be adjusted to unison with the generator; but the use of such receivers does not increase or diminish the necessity of observing the other conditions of efficient transformation; viz. that as little energy as possible should be used outside of the receivers, and that in the receivers as little energy as possible should be consumed except for the useful purpose. In any case the use of resonant receivers introduces a new difficulty, that of adjusting them to unison, and maintaining unison under the varying demands for energy by the consumers.

Now it seems to the writer that in the transmission of energy electrically by means of high frequency and high potential currents the difficulties in preventing losses are greatly increased; and although, by the use of such high potentials, light may be obtained by means of discharges through

rarefied gases without the accompaniment of so large a proportion of energy in the form of dark heat, yet the losses in the use of electrical energy in this form, and applied in this way, seem to much more than counterbalance the efficiency of the luminous source itself.

As has been already said, Mr. Tesla has devised some wonderfully brilliant and striking experiments illustrative of the action of high potential and high frequency currents, but when it comes to the application of these to artificial illumination he is silent upon the all-important question of efficiency, and from such information as we have, it would seem that very little progress had been made toward any reasonable result in that direction.

Part III. treats of miscellaneous inventions. These are of interest as showing the diversified character of Mr. Tesla's work. Some of them show great ingenuity, but probably there are few of them to which Mr. Tesla would to-day attach any practical value.

Part IV. relates to Mr. Tesla's World's Fair exhibit, and to the oscillator, which was made the subject of the lecture before the members of the Electrical Congress.

The personal exhibit was itself a very interesting one, showing as it did, the progress of Mr. Tesla's work, especially in the development of the rotating field apparatus. The reader will be glad to find embodied here brief but excellent descriptions of the separate pieces of apparatus there exhibited.

The oscillators shown by Mr. Tesla at the Congress lecture were very ingeniously designed, and would no doubt give a fairly uniform vibration under varying loads; but the claim that the vibrations would be so uniform under all conditions, that a clock actuated by the apparatus "would keep absolutely correct time" is probably not intended to be taken literally. The apparatus appears to be an excellent one for the production of alternating currents whose frequency can be accurately known, and it may serve an excellent purpose in experiments and investigations, and perhaps for telegraphy, but the writer is unable to feel that confidence in the great future of this apparatus that he felt in that of the rotating field motors when they were first explained to him in Mr. Tesla's laboratory.

The chapter devoted to the oscillators in this volume is very brief, and does not describe very fully the oscillators themselves, or the apparatus used in connection with them.

The work of Mr. Tesla described in this volume covers a period of hardly eight years. In that time the rotating field principle was developed, and a multitude of different forms of motors embodying the principle were built. The high frequency alternators, induction coils, condensers, lamps, vacuum tubes, etc., etc., have all been developed through successive

experimental failures, and the immense variety of apparatus shown in successful operation on the occasion of the three lectures on high frequency phenomena are only a tithe of the instruments and apparatus built. All this shows not only Mr. Tesla's wonderful fertility, but his industry and perseverance as well. The writer understands that it is wholly his own work; the few assistants employed by him in his laboratory being only mechanics who could give him no help in the development of the apparatus. Few men can point to more work accomplished in so short a time.

WILLIAM A. ANTHONY.

Drum Armatures and Commutators (Theory and Practice). By F. MARTEN WEYMOUTH. 8vo. 1 vol. London, The Electrician Publishing Co., 1894.

Mr. Weymouth has taken up, in the book before us, a specialized department of the subject of dynamo design and construction, and has treated it in a manner "entirely free from mathematics." It is to be questioned whether this method of treatment is a happy one in such a case as the one before us. A needless use of higher mathematics is certainly inadvisable, but the majority of men who are likely to read this book can readily understand and appreciate the comparatively simple formulas which may sometimes be used to replace sentences of explanatory matter. The author, in his introduction, calls this book an elementary one (or a beginner's book), but it is doubtful if a beginner could properly understand the theory of armatures as given by the text and figures of the first chapter, unless he had lost his title by previous careful study of the general theory of dynamos.

In the second and third chapters is given an explanation of the conditions existing in ordinary drum armatures, and the differences that mark windings for high and low pressure. Heavy windings are given much attention, and the difficulties met in winding armatures for large currents, with their attendant demand for compact and symmetrical end connections and good magnetic balance, are pointed out. These chapters are particularly full in the treatment of bar windings laid upon smooth drum armature cores. For the American reader this is somewhat unfortunate, since, in this country, bar windings are now commonly used only with toothed cores. The common surface winding made of wire receives comparatively little attention.

The next chapter is given up to a short discussion of ventilation and the prevention of Foucault or parasitic currents in bar conductors, but neither of these subjects is very fully treated. The usual methods of dividing the

conductors and twisting them to prevent excessive parasitic currents are well set forth. The cause of these currents is also examined, and is shown to be the irregular density of the magnetism under the polar tips. To remove this cause, and thus its effect, the suggestion is offered to make the air space greater under the pole tips. This construction is largely used in this country but for the principal object of gaining sparkless collection at the commutator. It serves this purpose well and at the same time does reduce parasitic currents, though it can scarcely be said to entirely remove their cause.

In five short chapters various forms of bar and special windings with their end connections are described. None of these have any extended use in this country, and many of them are unknown to service here. The author shows a satisfactory appreciation of the advantages of lathe-wound coils which may be made up in forms and be separately applied to the armature body. This section of the book is notable for the indistinctness of its winding diagrams which surely cannot be intended for beginners in the art. The presentation of poorly drawn diagrams of windings is a common fault of books dealing with this subject, and it is a fault for which no adequate excuse can be offered. The common forms of winding diagrams, which are used in the winding rooms of manufacturers, may be made perfectly clear if they are properly engraved.

The author does not mention toothed drum armatures which are meeting with so much favor with American makers, since their advantages have become fully appreciated and the means of avoiding their difficulties have been learned. Neither do multipolar windings receive the attention which their importance warrants.

Four excellent chapters are devoted to commutators and their constructive details. This section is well illustrated by drawings of commutators taken from the best English designers. The discussion of commutator insulation would lead us to believe that English makers are only beginning to realize that vulcanized fiber is not a satisfactory insulator for commutators. Successful American manufacturers have long since abandoned its use for this purpose, owing to its poor insulating and mechanical qualities when subjected to an atmosphere of varying humidity. The later chapters of the section, which deal in an excellent manner with the constructive details of commutators, tend to remove the erroneous impression regarding the use of fiber.

The latter half of the book is devoted to a consideration of the causes and the prevention of sparking. This section contains a great deal of interesting and useful information, but it is marred by prolixity, and in some parts by obscurity. Much of the discussion also appears to be impractical. Fortunately two important chapters (those on carbon brushes and armature

defects) stand out in contrast to the remainder of the book on account of their simple and practical explanations. In discussing carbon brushes the author limits the safe working current to thirty amperes per square inch. This will scarcely be concurred in by the majority of American manufacturers, who are in the habit of using densities of from forty to sixty amperes per square inch.[1] In his discussion of sparking and its prevention, the author properly considers the inductance of the armature coils, the electric pressure developed in each, and the magnetism cut by the short-circuited coils, as determining factors. In dealing with them as independent factors he falls into an error. It is now coming into general acknowledgment that the relations between these factors, rather than their actual proportions, determine the condition of sparkless operation.

Two chapters on armature reactions and their effects are based upon the well-known papers of Swinburne and Esson. Considerable later matter is also introduced from the familiar papers of Sayers and Fischer-Hinnen, but in no place is mentioned Ryan's arrangement of balancing coils for the prevention of field distortion.

Reviewed as a whole, this is a book that contains much of interest, but its value and interest to the majority of readers would have been increased by careful condensation of the less important sections. Unfortunately the book is not sufficiently thorough to give it the position amongst specialized technical works that its title might claim.

DUGALD C. JACKSON.

[1] Jackson's text-book on Electromagnetism and the Construction of Dynamos, p. 193.

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